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NON-LINEAR FORCES ON OSCILLATING PLATES: REVIEW AND ANALYSIS OF THE LITERATURE.

by

J. F./Dalzell

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The general objective of the present work was to look for alternatives to the conventional time domain model for the oscillatory forces on plates, using what data could be found in the literature. To the extent which information could be extracted from existing sources, an alternate to the conventional model can be synthesized, but only for sinusoidal plate or fluid motion. The results do not

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obviously point the way toward a real-time non-linear model suitable for more complex forms of motion . An approach for which there had been high initial hopes (the functional polynominal) appeared, at the end of the work, not to be so promising.

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NON-LINEAR FORCES ON OSCILLATING PLATES: REVIEW AND ANALYSIS OF THE LITERATURE

by

J.F. Dalzell

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CONTENTS

INTRODUCTION

The present work was stimulated in a quite indirect way by some obstacles encountered in studies of weakly non-linear ship motions, specifically ship rolling in random seas. The background will be summarized in the following paragraphs.

The overall purpose of research on ship motions is to enable improved quantitative predictions of real-world phenomena. In the ultimate practical application, the real-world ship motions problem must be considered to be stochastic; there seems no escape from this aspect of the problem. In the last two and one half decades the methods outlined by St. Denis and Pierson[1*] for the estimation of the magnitude of oscillatory ship motions in irregular seas have become firmly established in engineering practice. These methods apply strictly only to ship responses which can be assumed to be a linear function of wave height, and they involve as well the assumption that the wave process is Gaussian.

In this context the first recognized ship motions problem (ship rolling) remains a problem to some extent. Roll damping at least for low or zero ship speeds, has been considered to be a mixture of linear and quadratic damping for about a century.[2*] No modern hydrodynamic analysis has challenged the model--for that matter there appears to be no completely theoretically based prediction method for roll which is altogether free of empiricism with respect to roll damping. However, if the necessity for empiricism is accepted there is still another problem with roll and this is the simple fact that a non-linearity often appears to exist for low to moderate rolling amplitudes.

In the majority of applications to design what is most wanted with respect to rolling is a measure of the statistics of rolling maxima

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*1. St. Denis, M. and Pierson, W.J., Jr., "On the Motions of ships in Confused Seas, "SNAME Vol. 61, 1953.

*2. "The Papers of William Froude", The Institution of Naval Architects London, 1955

to be expected in various hypothetical or realistic irregular sea conditions for a variety of design alternatives. Ideally, what is needed (but not available) for the prediction of ship rolling statistics in random seas is a prediction framework analogous to the linear framework of St. Denis and Pierson[1] in which: a) at least weak non-linearities could be accomodated for the general multi-degree of freedom situation, b) multi-directional seas could be considered as input, c) the hydromechanic data required could be produced with conventional techniques, d) the prediction or a major part could be carried out in the frequency domain for economy (as well as to take advantage of the accumulating frequency domain descriptions of real sea waves) and e) the statistics of maxima could be estimated with firmly based theory in which the possible effects of non-linearities are accounted for.

In the absence of this ideal, modern practice involves linearization of the damping coefficient in some manner. With contemporary six degree of freedom ship motion algorithms the approach often taken is to linearize the roll damping coefficient and thereafter to conduct the analysis and make the irregular sea predictions as though the system was completely linear. Variations on this theme exist to the extent that it is possible to make iterative solutions so that the linearized damping coefficient is chosen to minimize errors in the final prediction for irregular waves.

Early fundamental work on the problem of a frequency domain prediction of non-linear rolling in irregular seas has centered upon the problem of predicting the spectrum and the variance of zero speed rolling, this being the case in which the damping non-linearity has been found to be most obvious, and the case for which it is plausible to reduce the problem to a single degree of freedom. Typically, a single degree of freedom rolling equation not far different from that of W. Froude[2] is solved in some sense for the random excitation case. The work of Kaplan[3*] and Vassilopoulos[4*] involve equivalent linerization techniques in the estimation

*3 Kaplan, P., "Lecture Notes on Non-Linear Theory of Ship Roll Motion in a Random Sea Way, ITTC Transactions, 1966.

*4 Vassilopoulos, L., "Ship Rolling at Zero Speed in Random Beam Seas with Non-Linear Damping and Restoration", Journal of Ship Research, Vol. 15, No. 4, December 1971.

of the variance of roll and the roll spectrum. In this technique non-linear elements are replaced by linear elements chosen so as to minimize the resulting mean square errors for the case of random excitation. Yamanouchi[5*] approached the problem with a perturbation technique for the solution to the non-linear differential equation. In this solution the spectrum of non-linear rolling turns out to be the sum of the linear roll spectrum and various convolutions of the linear roll velocity spectrum.

For practical purposes the result of both the above approaches is the predicted roll variance. The implicit assumption is that the statistics of maxima are adequately described by the Rayleigh distribution with parameter equal to square root of variance. This assumption has been partially vindicated by a study[6*] in which numerical simulation of the single degree of freedom rolling equation was carried out. The results indicated that while the Rayleigh distribution is probably not a completely proper assumption, reasonably good predictions under this assumption could be expected for quantile averages up to average of 1/10 highest amplitudes, despite inclusion of non-linearities within the nominal range of magnitude observed in unstabilized ships and models.

There are presently three additional approaches to the problem of the prediction of the statistics of non-linear random processes, time domain simulations (Monte-Carlo Methods), the Fokker-Planck Equation Method (Caughy[7*]) and the Functional Series approach (Wiener[8*], Barrett[9*],

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5* Yamanouchi, Y., "On the Effects of Non-Linearity of Response on Calculation of the Spectrum", ITTC Transactions, 1966.

6* Dalzell, J.F., "A Note on the Distribution of Maxima of Ship Rolling", Journal of Ship Research, Vol. 17, No. 4, December 1973.

7* Caughey, T.K., "Derivation and Application of the Fokker-Planck Equation to Discrete Non-Linear Dynamic Systems Subjected to White Noise Random Excitation", Journal of the Acoustical Society of America, Vol. 35, No. 11, November 1963.

8* Wiener, N., "Non-Linear Problems in Random Theory", The Technology Press of MIT and John Wiley and Sons, Inc., 1958.

9* Barrett, J.F., "The Use of Functionals in the Analysis of Non-Linear Physical Systems", Journal of Electronics and Control, Vol. 15, No. 6, December 1963.

Bedrosian[10*]). Neither of the last two can be considered as having been reduced to common practice.

The basic attraction of the time domain simulation approach is that if the system can be described in terms of differential equations, a numerical solution for a particular realization of the excitation can be achieved. The approach is definitely within state-of-art. There are two related difficulties. First, if the non-linear system is extremely complicated, the analyst is lucky to achieve a computer simulation which runs as fast as real time. If very long samples are required the approach can be prohibitively costly. Secondly, long samples are required if good statistical information is to be generated. Fundamentally, what is done in a Monte-Carlo Analysis is to generate large time domain samples, extract the required statistical information, derive the probable sampling errors, and then discard the most costly part of the procedure. This is the essential reason that the St. Denis and Pierson[1] frequency domain approach to the linear case has been so universally adopted--it is much faster and there are no crucial sampling problems.

The attraction of the Fokker-Planck approach is that in principle the statistics of the output are solved for directly. Haddara[11*,12*,13*,14*] uses the approach to derive expressions for the variance of roll for the

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10* Bedrosian, E. and Rice, S.O., "The Output Properties of Voltera Systems (Non-Linear Systems with Memory) Driven by Harmonic and Gaussian Inputs", Proceedings of the IEEE, Vol. 59, No. 12, December 1971.

11* Haddara, M.R., "On Non-Linear Rolling of Ships in Random Seas", International Shipbuilding Progress, Vol. 20, No. 230, October 1973.

12* Haddara, M.R., "A Modified Approach for the Application of Fokker-Planck Equation to the Non-Linear Ship Motions in Random Waves", International Shipbuilding Progress, Vol. 21, No. 242, October 1974.

13* Haddara, M.R., "A Study of the Stability of the Mean and Variance of Rolling Motion in Random Waves", International Conference on Stability of Ships and Ocean Vehicles, University of Strathclyde, Glasgow, Scotland, March 1975.

14* Haddara, M.R., "On the Stationary Coupled Non-Linear Ship Motion in Random Waves", International Shipbuilding Progress, Vol. 23, No. 262, June 1976.

case that the excitation spectrum is white (flat), in some cases finds it necessary to replace the quadratic damping with a cubic to facilitate the analysis, but does not directly derive the statistics of response. Evaluation of the statistics of response according to this approach are extremely difficult when the excitation spectrum is not white. Dello-Stritto[15*] has come close to accomplishing this, but it was necessary in his work to replace the non-analytic quadratic damping term with one of cubic form.

There are several attractions of the functional series approach. Among these are that as a conceptual framework the model is suitable for any reasonably well behaved wave input (regular, transient, or random), and since it contains the completely linear system as a special case it appears to have the potential of being a logical extension to present practice. In addition, theoretical prediction methods for spectra may be derived, and it appears that it may be possible to approximate the statistics of maxima. Finally, it is possible in principle to relate the functions required by the model to the results of hydromechanical analyses and experiment. Apart from complexity, there is a serious mathematical obstacle in applying the approach to the conventional ship rolling equation. This was pointed out some time ago by Vassilopoulos[16*]. It is that if the functional expansion and a differential equation are to be related, it appears that the equation must be analytic for small values of the variables. This is not the case for the "quadratic" term ordinarily used to represent the damping non-linearity. Some degree of success in applying this approach to single degree of freedom rolling was obtained in[17*] by replacing the quadratic damping with a cubic term.

15* DelloStritto, F., "An Analytical Approach to Non-Linear Ship Roll in a Random Sea", PhD Dissertation, Stevens Institute of Technology, 1978.

16* Vassilopoulos, L.A., "The Application of Statistical Theory of Non-Linear Systems to Ship Motion Performance in Random Seas", International Shipbuilding Progress, Vol. 14, No. 150, 1967.

17* Dalzell, J.F., "Estimation of the Spectrum of Non-Linear Ship Rolling: The Functional Series Approach", SIT-DL-76-1894, Davidson Laboratory, Stevens Institute of Technology, May 1976, AD A031 055/7GI.

There is a common thread running through most of the cited work. It is that the quadratic roll damping form, which has been accepted for the last century, is a serious analytical obstacle which needs to be overcome if improvements in the prediction of roll in irregular seas are sought. The form is awkward in perturbation analyses (Flower[18*] dispensed with it in his recent work), it apparently makes the full exploitation of Fokker-Planck methods next to impossible, and is inadmissible in the Functional Series approach. It seems permissible to speculate that the quadratic damping form may pose as much of a problem to alternate, better (but as yet unformulated) approaches.

No particularly conclusive general theory has been advanced for the inclusion of quadratic damping in the description of ship roll dynamics. The representation seems to have arisen as an intuitive extrapolation of real-fluid steady state drag theory to the unsteady case.

The validity of and justification for replacing quadratic roll damping by a linear plus cubic representation was addressed in[19*] (a slightly shortened version of the first part of[20*]). As near as could be found out, nearly the entire justification for the linear plus quadratic representation for roll damping is that it appears to work empirically--in the sense that curves of roll decay may be reasonably well fitted under this assumption. The results in[19] show that a linear plus cubic representation works just as well or better, at least within the limits of roll angles achievable in ship sallying experiments and within data scatter. If there are two analytical models which fit the observable data with roughly the same magnitude of error, the choice between the two must be

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18* Flower, J.O., "A. Perturbational Approach to Non-Linear Rolling in a Stochastic Sea", International Shipbuilding Progress, Vol. 23, No. 263, July 1976.

19* Dalzell, J.F., "A Note on the Form of Ship Roll Damping", Journal of Ship Research, Vol. 22, No. 3, September 1978.

20* Dalzell, J.F., "A Note on the Form of Ship Roll Damping", SIT-DL-76-1887, Davidson Laboratory, Stevens Institute of Technology, May, 1976, AD-A031 048/2G1.

made on bases other than the fit itself. In absence of such other considerations, the models may be considered equally good within the limitations of observable data--outside the range of data both are extrapolations. Physically, it is expected that the roll damping function be odd in roll velocity and positive for positive roll velocity. Both the mixed quadratic and cubic models can be made to fit this criterion through choice of coefficients. The historical preference for the quadratic model is apparently very simple; there has been no need to consider alternatives to a plausible and usable empirical concept.

All of the known attempts at improvement of prediction of roll statistics in irregular seas involve the assumption of differential equations and non-linearities of quite specific form. In view of the nature of the justifications advanced for the particular forms chosen, it seemed fair to speculate that the refinements possible in any of the noted statistical approaches may over-reach the validity of the assumed physical model when rolling throughout the range of practical interest is considered, and that some more attention paid to the fundamental physical nature of the non-linearities and their coupling with other modes of motion might be more profitable than the further work on the statistical methods themselves.

The foregoing considerations were the direct stimulus of the present work. Since ships with bilge or bar keels usually have the strongest evidence of non-linearity in their roll decrement curves, it has appeared reasonable to consider the unsteady forces on oscillating plates as fundamental to the roll problem. In investigations of forces on oscillating plates which were initiated in this connection (Martin[21*], Ridjanovic[22*]) the results were derived on the basis of the decay of amplitudes of a

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21* Martin, M., "Roll Damping Due to Bilge Keels", PhD Dissertation, State University of Iowa, June 1959.

22* Ridjanovic, M., "Drag Coefficients of Flat Plates Oscillating Normally to their Planes", Schiffstechnik, Bd 9 - Heft 45, 1962.

pendulum upon which the plate was fixed, and the quadratic damping assumption was implicit in the final analyses. Data generated in the latter reference is used in contemporary application, Cox[23*]. The fundamental nature of the oscillating plate problem is implicit in 24*, to cite an additional example.

In a broad sense, the quadratic model is by no means restricted to ship rolling. Indeed the Morison[25*] approach for forces on fixed piles is exactly this and is extensively used in Ocean Engineering. Keulegan and Carpenter[26*] undertook a laboratory investigation of forces on cylinders and plates in an oscillating fluid with the objectives of clarifying the accuracy of the Morison predictor equation, and of attempting a correlation of mean experimentally determined drag coefficients. It is evident (Wiegel[27*]) that there is exactly the same amount of theoretical justification for the Morison equation as there is for the quadratic roll damping model. The Morison equation is an intuitive model which is extensively used to produce useful engineering results.

An initial examination of the force data of[26] was made in[20] to see how well the quadratic component in the Morison equation was reflected in the actual data. It appeared from the analysis of[20] that the quadratic model for forces on an object in oscillatory flow has no particular magic. In fact it appeared that the use of the model could at times create apparent deviations from observation. The analysis also implied that a mixed linear plus cubic model might also fit the data for oscillatory forces.

23* Cox, G.G. and Lloyd, A.R., "Hydrodynamic Design Basis for Navy Ship Roll Motion Stabilization", Trans. SNAME, Vol. 85, 1977.

24* "Studies on Developing Accuracy in the Estimation Methods of Ship Propulsive Performance in a Seaway". Report of Ship Research Project No. 161, Japan, March 1976.

25* Morison, J.R., et al, "The Force Exerted by Surface Waves on Piles", Petroleum Trans., Vol. 189, 1950, pp 149-157.

26* Keulegan, G.H. and Carpenter, L.H., "Forces on Cylinders and Plates in an Oscillating Fluid", NBS Report 4821, Sept.1956; Journal of Research of the National Bureau of Standards, Paper No. 2587, Vol. 60, No. 5, May 1958.

27* Wiegel, R.L.,"Oceanographical Engineering", Prentice-Hall, Inc.,1964.

This finally completes the exposition of the path leading from the problem of prediction of roll in irregular seas to considerations of the forces on oscillating plates.

It is realized that forces on isolated plates in oscillatory motion do not by themselves provide even a partial picture in-so-far as ship roll damping is concerned; interactions between bilge keel and adjacent hull are to be expected, etc. If the primary purpose of the present work had been to develop directly applicable numerical estimates, a physical situation much closer to that of an installed bilge keel would have been considered. However, since the interest in the present work is in the form of things, not necessarily the quantitative description, it was thought vital as a first step to consider only the simplest physical situations and the most fundamental of data.

The overall concern in the present work was thus with examining alternatives to general concepts which have long been in use. The previous review of data concerning plates in oscillatory flow suggested that there may be alternatives to the conceptual quadratic model for the oscillatory forces. The immediate purpose of the present work was to continue this review using that fundamental data which is already available.

SCOPE OF THE DATA OF INTEREST

Since only qualitative theories appeared to exist for the unsteady flow about bluff bodies, it was important as a first step to consider only the simplest situations, lest too many complications prevent progress. On the other hand, the possibility that suitable data on oscillating plates would be extremely sparse had to be taken into account.

Since the form of the force response was the item of interest, it was decided to consider data wherein either the plate or the fluid was in motion (but not both). To avoid the complication of lift, interest was restricted to fluid or plate motion in a direction normal to the plane of the plate. Under these circumstances plates of any geometric aspect ratio in oscillatory flow would be of interest if the nominal flow was spatially uniform in the immediate vicinity of the plate. Similarly, plates attached normal to a splitter plane were considered of interest so long as overall flow or motion was parallel to the splitter plane. Finally, to emphasize the unsteady response and remove a complication of considerable significance, interest was restricted to cases in which the mean flow, or the mean motion, of the plate was zero.

It appeared prior to any serious attempt at literature searching that a great deal more fundamental unsteady flow experimental work has been done with circular cylinders than with plates owing to the importance of circular cylinders as elements of ocean structures. It was thus also of interest to examine recent work of this nature. In accordance, the scope of data was enlarged to include circular cylinders in oscillatory flow normal to their axes, or oscillating in a fluid in a direction normal to their axes, with the same restrictions upon flow as previously mentioned for plates.

Effectively the restrictions imposed upon the data imply three types of experimental situations: 1) A stationary plate or cylinder positioned at the node of a standing wave and parallel to it as in Keulegan[26], 2) A stationary plate or cylinder in an artificially induced plane oscillatory flow as might be done in a U-tube, 3) Submerged plates or cylinders mechanically oscillated.

The restrictions exclude the most prevalent situation for circular cylinders wherein the cylinder pierces the fluid free surface and oscillatory flow is induced by propagating surface waves past it, as well as all forms of hydroelastic response, arrays of elements, and all cases in which there was a mean flow.

LITERATURE SEARCH

Though a few sources of fundamental data on forces on oscillating plates and cylinders were known at the outset, it was thought important to try to make the study as inclusive as possible. Toward this end both a manual and an automated literature search was carried out.

The details of the procedures followed and the initial stages of the elimination of inapplicable references is detailed in Appendix A. Though in principle the search encompassed approximately four million citations in the literature, the foregoing restrictions on the type of data of interest resulted in only a relative handful of experimental references which could be of conceivable use in the present study. To be specific, the search ended with 14 experimental references having to do with unsteady forces on plates, 13 involving experiments on oscillating forces on cylinders, and six or eight analytical references.

In general, it appears that little or no fundamental experimental work on oscillating plates has appeared in the English literature since 1971, and that the purely analytical problem has remained as intractible as it was two decades ago when Keulegan and Carpenter[26] completed their experiments.

DIMENSIONAL ANALYSES

In the initial reviews of those references which were considered of conceivable use in the present effort there appeared to be a great variety of non-dimensional forms of dependent and independent variables used both for thinking and correlation purposes. Though it was realized that part of this situation was likely to be simple differences in notation, there seemed some point in making the first step in the present study a general review and correlation of non-dimensional forms rather than of data.

The purpose of the present work is to try to look at the problem from alternate points of view. All previous work in the field of current interest has involved dimensional analyses explicitly or implicitly. The reason for embarking upon the present analysis was not that previous analyses were considered incorrect, but that for any given non-trivial problem there exist an almost unlimited number of valid sets of dimensionless groups. Which particular set of dimensionless groups are employed depends as much upon the analyst's pre-conceived notions as upon the problem. The objective was thus to see if there really were dimensionally divergent points of view in the literature.

The point may possibly be clarified by noting that in a given problem the first step in an analyses is make up a list of all variables and parameters which influence matters. This step is under complete control of the analyst and depends entirely upon his understanding of the problem. Further, suppose that a valid set of dimensionless variables is developed from the specified list. The formal mathematics of dimensional analysis shows that so long as the operations are carried out one at a time:

1) Any member of the set of dimensionless variables may be replaced by itself raised to any non-zero power; and
2) Any member of the set of dimensionless variables may be replaced by itself multiplied by any or all of the other members of the set, each raised to any power.

In general, while two analysts performing a systematic dimensional analysis of the same set of variables may come up with valid sets of dimensionless groups having different form, they will each have the same number of groups,

and it will be possible to transform the results of the first analyst into those of the second; in other words, the formal methods of analysis imply that alternate sets of dimensionless groups are not independent and hence no one of them can be expected to contain any essential information not common to the others. The differences between the results of different analyses arise only when the formalities are over; correlations of empirical data may appear quite different when done according to different sets of dimensionless parameters.

The problem at hand has been considered in all the cases reviewed to involve unsteady, relatively low velocity flow of a fluid in the general vicinity of a rigid body. No thermodynamic, magnetic, elastic, or electrical effects are considered. As a consequence, the problem is assumed to have four dimensional categories; Force, Mass (M), Length (L), and Time (T) which are related through Newtons second law, so that three of these four categories are considered to be independent, basic dimensions. In the present case mass, length and time were taken to be the three independent dimensions.

The geometry of the present problem is conventionally denoted by a characteristic length which is taken to be the diameter of the cylinder or width of a plate. For the two-dimensional flow situation this is the only constant geometric length parameter. All analyses reviewed involve the assumptions that the fluid is incompressible and viscid, and that there is no free surface. As a consequence the gravitational field is not considered to be an important parameter, and the fluid itself is considered to be completely defined by a mass density and kinematic viscosity.

The last paragraph defines three parameters which seem always to be taken as constants in problems of the present type. There seems no argument about their general relevance. The three are summarized as follows:

Quantity	Symbol	Dimension
Characteristic Width or Diameter	D	L
Fluid Mass Density	ρ	M/L^3
Fluid Kinematic Viscosity	ν	L^2/T

It may be noted that the dimensions of these three variables contain each basic dimension at least once, that no pair of the three have the same dimensional form, and that it is not possible to form a dimensionless group from these three variables alone. These are the conditions required for the three variables to be a valid choice of repeating variables in a dimensional analysis. Assuming three basic dimensions, the simplest way to produce a valid set of (N-3) dimensionless variables from a list of N is to pick three "repeating" variables satisfying the above conditions and use them consistently to non-dimensionalize the remaining variables by inspection.

For the purposes of the present analysis the foregoing variables will be chosen to be repeating. In the context of what appears in the literature, this choice is rather un-conventional. These three variables are really just constant parameters for any given physical experimental situation. Once the experiment is set up they are fixed, and do not vary with time or with any of the other independent variables of the problem. By picking these three variables no pre-supposition of the form of the result is made; this being the general objective of the present analysis.

To complete the formal analysis a general (and redundant) list of additional variables of possible consequence is written. These appear in the left half of Table I. First on the list is the only dependent variable of interest, the unsteady force (F) on the plate or cylinder. The next three items on the list involve time: "t" denotes on-going time; "T_m" and "ω" denote characteristic constants having time and frequency dimensions. In a similar fashion W_m, U_m and A denote characteristic constants having acceleration, velocity and displacement dimensions respectively. The time dependent displacements of the oscillating plate or cylinder from its mean position are represented by X(t), and the first two time derivatives of X(t) are also included. (X(t) may alternately represent the motion of a fluid particle if the plate or cylinder is stationary). Finally, to be complete, other fixed geometric parameters (length, thickness of plate, etc.) are denoted by d_j.

TABLE I

List of Variables and Dimensionless Groups
(Repeating Variable D, ρ, ν)

Quantity	Symbol	i	Γ_i	Π_i
Force (Dependent Variable)	F	1	$F/\rho\nu^2$	$F/\rho\nu^2$
Time Constant	T_m	2	$T_m\nu/D^2$	$T_m\nu/D^2$
Frequency, Circular	ω	3	$\omega D^2/\nu$	$\omega T_m/2\pi$
Time (ongoing)	t	4	$t\nu/D^2$	t/T_m
Acceleration Constant	W_m	5	$W_m D^3/\nu^2$	$W_m D^3/\nu^2$
Acceleration (Function of Time)	$\ddot{X}(t)$	6	$\ddot{X}(t)D^3/\nu^2$	$\ddot{X}(t)/W_m$
Velocity Constant	U_m	7	$U_m D/\nu$	$U_m D/\nu$
Velocity (Function of Time)	$\dot{X}(t)$	8	$\dot{X}(t)D/\nu$	$\dot{X}(t)/U_m$
Displacement Constant	A	9	A/D	A/D
Displacement (Function of Time)	$X(t)$	10	$X(t)/D$	$X(t)/A$
Other Fixed Geometric Parameters	d_j	11	d_j/D	d_j/D

The eleven dimensionless groups resulting from the use of the repeating variables D, ρ, and ν to non-dimensionalize the list of remaining variables are shown in Table I under the heading "Γ_i". So long as the variables are all considered quantities necessary to express the physical relationships involved, the conceptual relationship between the variables may be written:

$$f(\Gamma_1,\Gamma_2 \ldots \Gamma_{11})=0$$

This is essentially the end of the formal analysis.

Once having a consistent set of dimensionless variables, arbitrary manipulations of the set may be performed according to the rules previously noted. The column labelled "Π_i" is the result of such a completely arbitrary manipulation. Variables Γ_3 and Γ_4 are replaced using Γ_2; Γ_6 is replaced by use of Γ_5; and Γ_8 is replaced by use of Γ_7.

Thus far, nothing has been said about which ones the independent variables are, or what the various characteristic constants mean; the variables noted could apply to practically any incompressible fluid flow problem. There is one glaring redundancy in the Γ_i list in Table I. It is that $X(t)$, $\dot{X}(t)$ *and* $\ddot{X}(t)$ are mathematically related. In terms of dimensionless groups, Γ_6 and Γ_8 are derivatives of Γ_{10} with respect to Γ_4. Thus only one of the three is really needed. If $X(t)$ (and t) are considered the independent variables, and the various characteristic constants are assumed to be unrelated, the essential functional relationships could be written:

$$F/\rho\nu^2 = f_1(X(t)/D, t\nu/D^2)$$

The above form does not really help much with the present problem (which is the examination of existing data). All of the fundamental data in hand involves simple harmonic motion of the plate or cylinder; or of the fluid. To specialize the analysis to this case let the motion of the plate be:

$$\begin{aligned} X(t) &= A \sin 2\pi t/T_m \\ \dot{X}(t) &= (2\pi A/T_m) \cos 2\pi t/T_m \\ \ddot{X}(t) &= -(2\pi/T_m)^2 A \sin 2\pi t/T_m \end{aligned} \qquad (1)$$

and define:

$$U_m = 2\pi A/T_m$$

$$W_m = (2\pi/T_m)^2 A$$

$$\omega = 2\pi/T_m \qquad (2)$$

The harmonic assumption allows specific definition of the arbitrary constants assumed. T_m is the period of the motion, A the displacement amplitude, U_m the velocity amplitude, W_m the acceleration amplitude and ω is the circular frequency corresponding to T_m.

Substituting the above relationships into the Π_i, Table I, it may be observed that:

$$\Pi_3 = 1$$

$$\Pi_5 = (2\pi)^2 \; \Pi_9/\Pi_2^2$$

$$\Pi_6 = -\sin 2\pi\Pi_4$$

$$\Pi_7 = 2\pi\Pi_9/\Pi_2$$

$$\Pi_8 = \cos 2\pi\Pi_4$$

$$\Pi_{10} = \sin 2\pi\Pi_4$$

Since Π_5 and Π_7 may be replaced by pure constants by utilizing Π_2 and Π_9, and Π_6, Π_8 and Π_{10} are functions of Π_4 all of these terms in the original analysis, Table I, are redundant under the harmonic motion assumption. The basic result of the analysis for harmonic motion becomes:

$$F/\rho\nu^2 = f_3 \; (\Pi_2, \Pi_4, \Pi_9, \Pi_{11})$$

$$= f_3 \; (T_m\nu/D^2, \; t/T_m, \; A/D, \; d_j/D) \qquad (3)$$

In a roundabout way the analysis has come down to exactly what must be expected in an experiment involving harmonic inputs or excitation. Exactly three independent variables completely define the input; period, phase and amplitude.

In one respect Equation 3 is quite unlike any of the similar expressions in the literature. This is the form of the dimensionless group involving the dependent variable. If this group is multiplied by $\left(\Pi_2^2 \; \Pi_9^{-2} \; \Pi_{11}^{-1}\right)$ however, the result is:

$$\frac{(2\pi)^2 \; F}{\rho D d_j \; U_m^2}$$

(after substituting $U_m = 2\pi A/T_m$). The dependent variable in Equation 3 may be replaced by the product of the above expression and any pure constant, so that Equation 3 becomes:

$$F/\rho U_m{}^2 D d_j = f_4(T_m\nu/D^2, t/T_m, A/D, d_j/D) \qquad (4)$$

The form of the dependent variable in Equation 4 is that uniformly adopted in the literature, apart from minor differences such as replacing ρ by $\rho/2$, and the assignment of d_j which is usually the length of the plate or cylinder, and is sometimes absorbed in F to produce force per unit length.

Considering the groups within the right hand side of Equations 3 or 4 it may be noted that under the harmonic motion assumption:

$$2\pi\frac{A}{D} = \frac{U_m T_m}{D} = \text{"The Keulegan-Carpenter Number"}$$

Thus by pure substitution the A/D term in Equations 3 or 4 may be replaced by $2\pi A/D$ or by $U_m T_m/D$. All three forms appear in the literature with "D" sometimes defined as a half width. When the $2\pi A/D$ form is used it is often denoted "K" or N_k for "The Keulegan-Carpenter Number".

The first term on the right hand side of Equation 3 or 4 is seen in at least three different forms which are trivial manipulations:

$$1/(T_m\nu/D^2) = D^2/T_m\nu$$

$$1/\sqrt{T_m\nu/D^2} = D/\sqrt{T_m\nu}$$

$$2\pi/(T_m\nu/D^2) = \omega D^2/\nu$$

The most common form of the first right hand term of Equations 3 or 4 is obtained by inverting it and multiplying by 2π times the third term:

$$2\pi(D^2/T_m\nu)\ (A/D) = 2\pi AD/T_m\nu$$

$$= U_m D/\nu$$

This last form is unformally called the "Reynolds Number" because of its similarity in form to the steady flow Reynolds Number.

The phase term in Equations 3 or 4 takes care of temporal variation within one period by the harmonic assumption. In the few cases where something other than (t/T_m) is explicitly noted, the term is effectively

replaced by sines or cosines of $(2\pi t/T_m)$. For example, under the harmonic assumption: two variations were noted in the literature:

$$X(t)/A = \sin 2\pi t/T_m$$

and:

$$\frac{\dot{X}(t)D}{\nu} = \left(\frac{U_m D}{\nu}\right) \cos 2\pi t/T_m$$

It was thus seen that all of the non-dimensional forms used in the available references are derivable from the present analysis. This shows only that no analysts have made fundamentally different assumptions about what is important. For harmonic oscillation all seem to come down to amplitude, period, and phase parameters, and implied strict geometrical similitude.

Correlations of the various forms with one-another indicate that in the literature there are about six alternate permutations of the non-dimensional forms obtained by various investigators; three of which are dominant. These alternate permutations are summarized in Table II, and the essential form of the present generalized analysis is included as a seventh. In all cases, the geometric parameter (d_j/D) is assumed to be the same, present notation is utilized, and all pure number factors are omitted.

The first form shown in the Table is that of Keulegan and Carpenter[26]. They evidently (at least initially) considered U_m, to be the "amplitude", and used it as one of their repeating variables. Accordingly, they called $(U_m T_m/D)$ the "period parameter" and (by elimination) considered the unsteady Reynolds Number to be the amplitude parameter. The second form shown in the Table takes account of the mathematical equivalence of $(U_m T_m/D)$ and $(2\pi A/D)$ and indicates the amplitude parameter as A/D and the unsteady Reynolds Number as the "period" parameter". Either Form 1 or Form 2 are derived, or noted, as the results of dimensional analysis in the bulk of the literature.

Essentially, the unsteady Reynolds Number $(U_m D/\nu)$ of Form 2 is a mixed parameter; both amplitude and period are involved. It is of considerable importance in controlling experiments to have each independent variable which can be regulated occur only in one of the independent dimensionless parameters. Thus if the amplitude ratio (A/D) is considered of prime

TABLE II

Alternate Forms of the Results of Dimensional Analysis

Form	Force	Amplitude	Period	Phase
1	$F/\rho U_m{}^2 Dd_j$	$U_m D/\nu$	$U_m T_m/D$	t/T_m
2	"	A/D	$U_m D/\nu$	"
3	"	"	$D^2/T_m\nu$	"
4	"	"	$\omega D^2/\nu$	"
5	"	"	$U_m D/\nu$	$X(t)/A$
6	"	$U_m T_m/D$	$D/\sqrt{\nu T_m}$	$\dot{X}(t)D/\nu$
7	$F/\rho\nu^2$	A/D	$\omega D^2/\nu$	t/T_m

importance (one main conclusion of Keulegan and Carpenter[26]) it follows that it ought to be eliminated from the period parameter of Form 2. Forms 3 and 4 in Table II arise from just such a replacement operation utilizing the non-dimensional independent variables of Form 2. Form 4 differs from Form 3 only in that circular frequency of excitation is explicitly noted. Though possibly not the first to make this transition, Sarpkaya[28*, 29*] advance something beyond the above for the period parameter of Form 3. He calls it the frequency parameter, denotes it by β, and notes that the same parameter is important in laminar boundry layer theory. He notes further that in the case of oscillations of a cylinder without separation in a fluid otherwise at rest, the unsteady forces should be uniquely determined in terms of β. This seems quite an important observation in the present context since it implies that the organization of the independent variables implied by Forms 3 and 4 would be valid as $A \to 0$; that is, for the linear case, Tuck[30*].

Forms 5 and 6 of Table II are rather isolated instances. Form 5 is that of Garrison [31*], and as has been shown, is fundamentally no different than Form 2. Form 6 (Dalton[32*]) is essentially Form 3 with the phase parameter multiplied by the unsteady Reynolds Number. This latter approach was developed in support of attempts to represent force on a cylinder as a continuous function.

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*28. Sarpkaya, T., "In-Line and Transverse Forces on Cylinders in Oscillatory Flow at High Reynolds Numbers", Journal of Ship Research, Vol. 21, No. 4, December 1977.

*29. Sarpkaya, T., "The Hydrodynamic Resistance of Roughened Cylinders in Harmonic Flow", The Naval Architect, March 1978, Transactions RINA, Vol. 120, 1978.

*30. Tuck, E.O., "Calculation of Unsteady Flows Due to Small Motions of Cylinders in a Viscous Fluid", California Institute of Technology. Report 156-1, December 1967, AD-829877.

*31. Garrison, C. J.Field, J.B., and May, M.D., "Drag and Inertia Forces on a Cylinder in Periodic Flow", Journal of the Waterway, Port, Coastal and Ocean Division of ASCE, Vol. 103, No. 2, May 1977. pp 193-204.

*32. Dalton, C., Hunt, J.P., and Hussain, A.K.M.F., "Forces on a Cylinder Oscillating Sinusoidally in Water--2. Further Experiments", Offshore Technology Conference Paper 2538, May 1976.

It may be noted that the independent variables of Equations 3 or 4 are the same as those of Form 3 in Table II, apart from an inversion of the period parameter. (One advantage for making the inversion is that the resulting numbers will be much larger than unity rather than much smaller). The amplitude/frequency form of the independent variables is chosen as the result of the present general analysis, Form 7, Table II. For the reasons just noted this form of non-dimensional independent variables seems most appropriate when alternate views of existing data are contemplated.

It has been noted that the form of the dependent non-dimensional variable uniformally adopted in the literature is that indicated for the first six forms in Table II. Only the result of the present analysis is different. The particular form of the non-dimensional variable uniformally adopted is that conventionally used for steady flow force coefficients. It appears that as far as dimensional reasoning is concerned, all work thus far has involved the hypothesis of the existence of either an averaged drag coefficient or of an "instantaneous" drag coefficient; in both cases of the conventional form for steady flow. The form of the dependent variable usually utilized is thus mixed, both amplitude and period are present in the denominator in the form of maximum velocity (U_m) squared. Because of the squared velocity, it is clear that all prior dimensional reasoning has been initially biased toward the quadratic model for unsteady forces.

The possible advantage of Form 7 of Table II in the present context is that none of the dimensionless groups are mixtures of independent and dependent variables. There is thus no initial bias as to the physical model, yet it has been shown that as far as dimensional reasoning is concerned there is nothing present in the conventional sets of variables not present in Form 7.

THEORETICAL ANALYSES

For practical purposes the summary by Keulegan and Carpenter[26] of the theoretical state of affairs is much more current than the date of the work would imply. The classical analytical work on the forces on bluff bodies is referenced as having been started by Stokes around 1850, both for the case of arbitrary motion and that of uni-directional time varying motion (Iverson[33*]). Very early work on accelerated motion indicated that the force on a bluff body depends, generally, upon the history of its acceleration as well as instantaneous values of velocity and acceleration, a point apparently confirmed by Iverson[33]. The suggestion that the flow has "memory" is consistent with current qualitative descriptions of large amplitude motion but in the common quantitative models the effects of memory are suppressed indirectly by considering steady state harmonic motion of fluid or body and allowing the coefficients which multiply instantaneous velocities and accelerations to be amplitude and/or time dependent. Keulegan and Carpenter[26] indicate that the form of the Morrison[25] equation which formed their point of departure (and that of virtually all other recent work) is in agreement with the results of Stokes for the force on a sphere oscillating in a viscous fluid.

Considering the two dimensional problem, and expressing the force per unit length on a cylinder or plate as (F/L) where L is the length of the cylinder, the model may be written as follows:

$$F/L = \tfrac{1}{4}\rho\pi D^2 C_m \dot{U} + \tfrac{1}{2}\rho D C_d |U| U \qquad (5)$$

Where: U denotes instaneous velocity

C_d is an unsteady "drag" coefficient

C_m is an unsteady "mass" coefficient

The first term in Equation 5 is considered to be the "mass" term and the second the "drag" term because of the assumed dependency upon acceleration

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*33. Iverson, H.W., and Balent, R., "A Correlating Modulus For Fluid Resistance in Accelerated Motion", Journal Applied Physics, Vol. 22, No. 3, March 1951.

and velocity. The added mass and drag coefficients may be considered to variable in time, but what is most often sought is averaged constant values of these coefficients which result in reasonable approximations to observed data for harmonic motion or flow.

McNown[34*], [35*] developed an alternate model for the mass term, and his expression may be written:

$$F/L = \tfrac{1}{4}\rho\pi D^2 \left[\frac{d(kU)}{dt} + r\dot{U}\right] + \tfrac{1}{2}\rho D C_d |U| U \tag{6}$$

where: r is a component of a "true" constant added mass coefficient and arises fromthe pressure gradient in the ambient flow.

and:

k is a time-varying mass coefficient.

The form of Equation 6 has seldom been explicitly used because averaged values of C_m are usually sought.

In comparing results from experiments involving oscillatory flow with data from experiments where the plate or cylinder is oscillated, the influence upon the effective C_m of the pressure gradient is usually accounted for. Since the pressure gradient term is proportional to volume, "r" in Equation 6 become unity for the circular cylinder, so that C_m in Equation 5 is expected to be higher (by an additive term of unity) for cases in which the flow oscillates than in cases where the cylinder oscillates. On the other hand, the influence of the pressure gradient is expected to be nil in the case of plates in oscillating flow, Buchanan[36*],

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*34. McNown, J.S., "Drag in Unsteady Flow", Proceedings of IX International Congress of Applied Mechanics, Brussels, 1957, Vol. III, pp 124-134.

*35. McNown, J.S., and Keulegan, G.H., "Vortex Formation and Resistance in Periodic Motions", Proceedings of ASCE, Engineering Mechanics Division, January 1959.

*36. Buchanan, H., "Drag on Flat Plates Oscillating in Incompressible Fluids at Low Reynolds Numbers", NASA-TM-X-53759, July 1968, N69-17466.

at least for plates which are relatively thin. For the fundamental problem of present interest, the influence of the pressure gradient on the "mass" term of Equation 5 is generally considered the only difference between the case of oscillating fluid and that of the oscillating plate or cylinder, otherwise the kinematic aspects of the problems are the same.

One of the interesting aspects of the problem which was addressed early, McNown[35], and is still not completely understood (McKnown[37*]) is whether or not there is a relationship between the C_m and C_d of Equation 5. The data of Keulegan and Carpenter[26] suggests that there is some relationship, and McNown[34] advanced an analytical model which demonstrated the point. In general, the notion that an oscillating plate or cylinder is a physically causal system would suggest that there should be a relationship, at least to a linear approximation.

The qualitative interpretation of the force induced on oscillating bluff bodies has largely to do with the cyclic formation of vortices. When the amplitude of motion is very small there may be little or no vortex formation for the circular cylinder and symmetrical formation for the plate. As amplitude grows, vortices are shed assymetrically, and if the amplitude is sufficiently large a quasi-steady Karman vortex street is envisioned. Unfortunately, for applications of practical interest, the relative magnitude of amplitude of interest is in the range where only a few vortices (or even a fraction of a vortex) can be shed before the motion reverses, and in this situation the vortices shed on the previous half cycle may be in near-enough proximity to influence the force. Analytical approaches to the computation of the force are available if the strength and position of all the vortices are known, Sarpkaya[38*]. Unhappily, it appears that the

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*37. McKnown, J.S., and Learned, A.P., "Drag and Inertia Forces on a Cylinder in Periodic Flow", (Written Discussion of Reference 31), Journal of the Waterway, Port, Coastal and Ocean Division of ASCE, Vol. 104, No. 2, 1978, pp247-250.

*38. Sarpkaya, T., "Lift, Drag and Added Mass Coefficients for a Circular Cylinder Immersed in a Time Dependent Flow". ASME Journal of Applied Mechanics, Vol. 30, Series E, No. 1, March 1963, pp 13-15.

prediction of the strength and position of all the shed vortices is beyondpresent state of analytical art, though only those in the immediate vicinity of the body need be taken into account. (Sarpkaya[29], Stansby[39*]). Various lumped or discrete vortex models have been developed (Ward[40*] for example), but are apparently utilized in a largely qualitative way.

With respect to the form of the Morison Model, Equation 5, the early theoretical work suggested only that it is a plausible representation, and this situation seems not to have been changed by more modern efforts.

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*39. Stansby, P.K., "An Inviscid Model of Vortex Shedding From a Circular Cylinder in Steady and Oscillatory Far Flows", Department of Civil Engineering Report No. 77/84, May 1977; Proc Institution of Civil Engineers (London), Vol. 63, Part 2, December 1977, pp 865-880.

*40. Ward, E.G. and Dalton, C., "Strictly Sinusoidal Flow Around a Stationary Cylinder", ASME Journal of Basic Engineering, December 1969.

PRIOR REVIEWS OF EXPERIMENTAL RESULTS

The most recent prior review and attempt at correlation of basic data on oscillating plates and cylinders appears to be that of Tseng and Altman[41*] in 1968. This review was a preliminary to new experimental work involving a more complicated geometry so that their own experimental work lies outside the present scope of interest. In addition to the work of Keulegan[26], McNown[34,35], Martin[21] and Ridjanovic[22], experimental results by Brown[42*], Paape[43*], Henry[44*], and Woolam[45*] were discussed and attempts at correlation were made.

Tseng and Altman's review[41], though short, contributes several important items. Predating Sarpkaya's[28,29] almost exactly similar analysis by 8 years, they showed that Keulegan's[26] average drag coefficient data for circular cylinders was Reynolds Number dependent as well as amplitude dependent. No such conclusion was obtainable for flat plate data, but it was noted that an "edge effect" should be present for flat plates. Effectively the idea is that if sufficiently sharp plate edges are available, eddy generation may be maintained at very low amplitude ratios, thus compounding the problem of correlation of the low amplitude results of various investigations. Considering the results of all investigators reviewed, they made a conceptual division of averaged drag coefficient for all shapes into four regimes according to increasing magnitude of amplitude ratio:

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*41. Tseng, M. and Altmann, R., "The Hydrodynamic Design of Float Supported Aircraft, 1--Float Hydrodynamics", Report 513-5, Hydronautics, Inc. October 1968.

*42. Brown, P.W., "The Effect of Configuration on the Drag of Oscillating Damping Plates", Davidson Laboratory, Stevens Institute of Technolocy, Report 1021, May 1964.

*43. Paape, A., and Breusers, H.N.C., "The Influence of Pile Dimensions on Forces Exerted by Waves", 10th. Conference on Coastal Engineering, Tokyo, Chapter 48, p. 840, 1966.

*44. Henry, C.J., "Linear Damping Characteristics of Oscillating Rectangular Flat Plates and Their Effect on a Cylindrical Float in Waves", Davidson Laboratory, Stevens Institute of Technology, Report 1183, June 1967, AD-657636.

*45. Woolam, W.E., "Drag Coefficients for Flat Square Plates Oscillating Normal to Their Planes in Air, Final Report", Southwest Research Institute Report 02-1973, NASA CR-66544, March 1968, N68-17911.

1) Regime of viscous shear flow
2) Near Potential flow, no large eddies
3) Regime of decreasing eddy size and distance between eddy structure and body
4) Regime of progessive shedding of eddies

Because few independent determinations of added mass coefficients existed in the literature, Tseng and Altmann were unable to do much correlation, but noted that the values obtained by Paape[43] were generally divergent from those of Keulegan[26].

Finally, in their analysis, there was no motivation for Tseng and Altmann to question the basic conceptual form, Equation 5.

OVERVIEW OF THE AVAILABLE EXPERIMENTAL REFERENCES ON PLATES

Of the references initially thought to be of possible use in the present exercies (Appendix A) some were and some were not. The objective here is to look at the data from other points of view if possible. Accordingly, some semblance of the basic observations has to be available. When the results given are very compressed little can be accomplished.

Judging by the available English language accounts, the Japanese school of thought[24], Kato[46*], Tanaka[47*, 48*] appears to be thoroughly based in the quadratic model for forces on bilge keels, and is most often oriented toward development of empirical expressions for immediate application. The compression of results in Tanaka[47] for instance is so thorough that it is impossible to imagine how to examine them from any other point of view.

Nominally, parts of the work by Mercier[49*] and Gersten[50*, 51*] on scale effects on roll damping appeared interesting in the present context because model bilge keels installed on cylindrical sections were instrumented for force measurement during angular oscillations of the section. However, in neither work was the instrumentation fully capable of resolving the forces on a segment of bilge keel. Mercier[49] omits any analysis, and Gersten[50], presents only an indirect analysis showing that the quadratic assumption is sufficiently plausible for engineering purposes. Though the need for more fundamental studies on the dynamic forces upon oscillating plates was

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*46. Kato, H., "Effect of Bilge Keels on the Rolling of Ships", Memories of the Defence Academy, Japan, Vol. 4, 1966.

*47. Tanaka, N. and Kitamura, H., "A Study on the Bilge Keels (Part 2, Full Sized Experiments)" J. of Society of Naval Architects of Japan, Vol. 103, 1958.

*48. Tanaka, N., "A Study of the Bilge Keels, (Part 4 - On the Eddy-Making Resistance to the Rolling of a Ship Hull), Journal of the Society of Naval Architects of Japan, Vol. 109, 1960.

*49. Mercier, J.A., "Scale Effect on Roll Damping Devices at Zero Forward Speed, " Davidson Laboratory, Stevens Institute of Technology, Report 1057, February 1965.

*50. Gersten, A., "Roll Damping of Circular Cylinders With and Without Appendages", NSRDC Report 2621, October 1969.

*51. Gersten, A., "Scale Effects in Roll Damping", Proceedings of the 16th. ATTC, 1971.

expressed by Mercier[49], the basic objectives of these studies were scale effect on rolling rather than on the fundamental problem of present interest.

Eliminating the experimental references just discussed from further consideration, there appear to be just eleven distinct experimental references on oscillating plates. Eight of these were known to or discussed by Tseng and Altmann[41], the three references not considered by them being: Stephens and Scavullo[52*], Cole and Gambucci[53*]; and Shih and Buchanan[54*]. (Shih[54] is effectively a published version of Buchanan[36]).

The eleven references may be roughly classified into three groups according to the approximate range of experimental amplitude ratio considered. In the first group (Keulegan[26], Martin[21] and Ridjanovic[22]) amplitude ratios, A/D, range from about 1/2 to more than 10. In the second group (McNown[34], Cole[53], Paape[43] and Shih[54]) amplitude ratios range from about 1/5 to 2. Finally, the third group (Brown[42], Henry[44], Stephens[52] and Woolam[45]) is comprised of experiments wherein amplitude ratios range from 1/100 to about 1/4.

Both the technical purpose and the locally feasible experimental apparatus evidently influenced the choice of amplitude ratio. Brown[42] and Henry[44] were interested in hydrodynamic damping at relatively small amplitude ratios. The thrust of Stephens[52] and Woolam[45] was toward structural vibration of thin panels, thus the small amplitude emphasis. Cole[53] and Shih[54] were involved with damping of fuel motion in liquid fueled rocket boosters; the range of amplitude ratio chosen by them was

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*52. Stephens, D.G., and Scavullo, M.A., "Investigation of Air Damping of Circular and Rectangular Plates, A Cylinder and a Sphere", NASA TN D-1865, 1965.

*53. Cole, H.A. and Gambucci, B.J., "Measured Two-Dimensional Damping Effectiveness of Fuel Sloshing Baffles Applied to Ring Baffles in Cylindrical Tanks", NASA, TN D-694, 1961.

*54. Shih, C.C., and Buchanan, H.J., "The Drag on Oscillating Flat Plates in Liquids at Low Reynolds Numbers", J. Fluid Mechanics, Vol. 48, Part 2, 1971.

evidently reasonable in this application. The background to McNown's experiments[34] is not evident but it appears that his apparatus or the preliminary nature of his program limited amplitude ratio to about unity. In the remaining references (Keulegan[26], Martin [21], Ridjanovic[22] and Paape[43]) the emphasis was upon ocean engineering or ship rolling applications. It is evident that Paape[43] was apparatus bound and could not achieve amplitude ratios in excess of about 2.5.

There are essentially three types of experiments involved in the references: 1) Stationary plates positioned at the node of a standing wave (Keulegan[26], McNown[34]); 2) Forced Oscillation of a plate (Cole[53], Paape[43], Woolam[45], Shih[54]); 3) Plate mounted upon a passive oscillator of some type (Stephens[52], Woolam[45], Martin[21], Ridjanovic[22], Brown[42], Henry[44]). The last technique involves the analysis of the rate of decay of oscillations of the oscillator after release from a mechanically induced initial displacement, and the results are usually confined to estimates of some sort of averaged drag coefficient. Forces must be measured in the first two techniques. Henry[44] measured forces but analyzed them as decaying oscillations.

For purposes of a first overview at least, a rough estimate was made of the range of unsteady Reynolds Number ($U_m D/\nu$) achieved in each investigation. No great precision was possible for some of the experiments because maximum velocities (U_m) often had to be inferred. Where uncertainty existed, the available numbers were combined in such a way as to result in a probable over-estimate. The resulting ranges of amplitude and Reynolds Number achieved in each experiment are shown in Figure 1. Each of the boxes defining the two ranges is labelled with the appropriate reference number.

The classical work of Keulegan and Carpenter[26] has the narrowest range of Reynolds Number and the largest range of amplitude. It is notable that all subsequent experiments, save two, have Reynolds Number ranges centered upon that of Keulegan and Carpenter. The two exceptions are Cole[53] and Shih[54]; the two sets of experiments oriented toward fuel sloshing. The divengence from the norm in these cases resulted from the

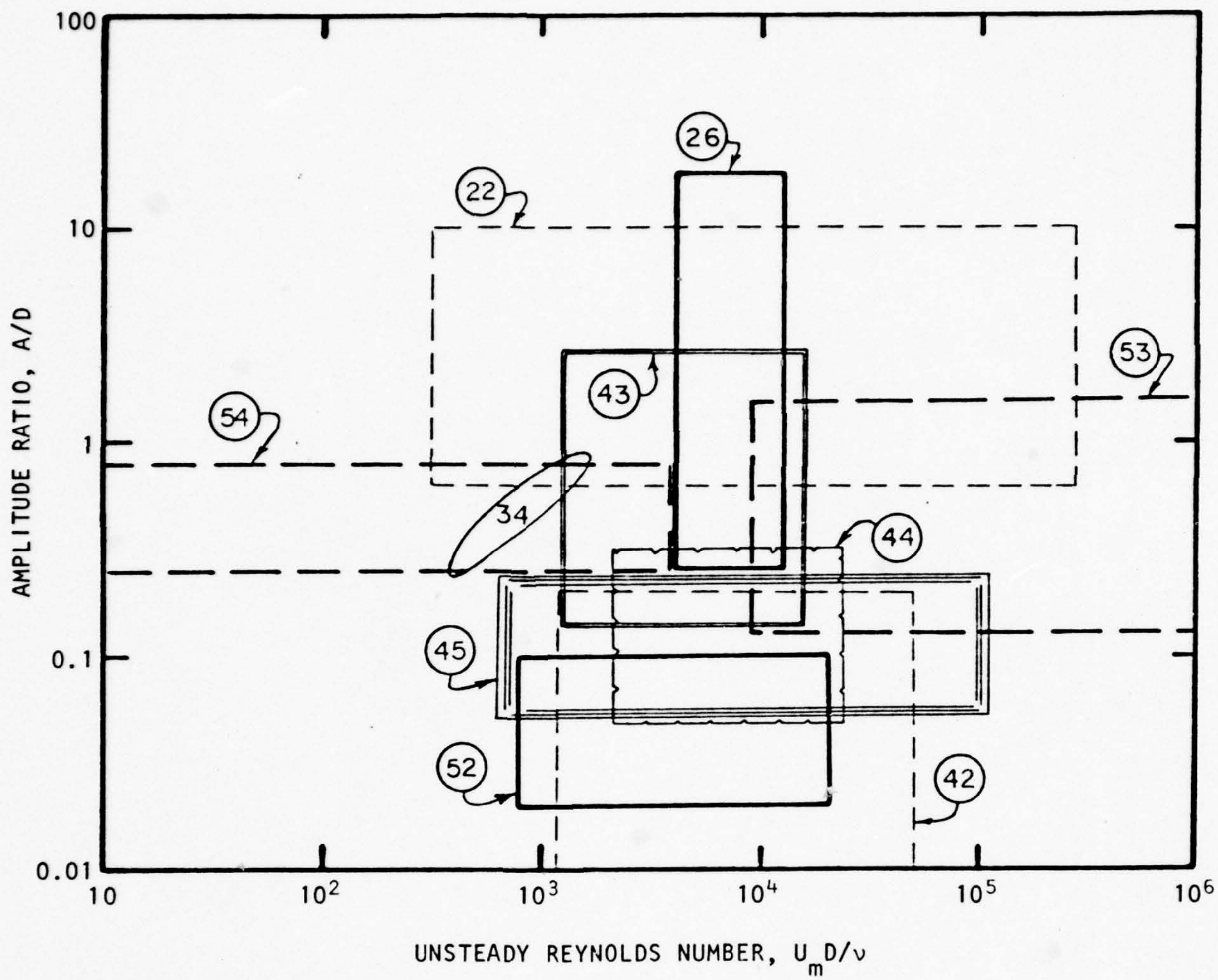

FIGURE 1 APPROXIMATE RANGES OF NON-DIMENSIONAL AMPLITUDE AND UNSTEADY REYNOLDS NUMBER ACHIEVED IN PLATE EXPERIMENTS, REFERENCES 22, 26, 34, 42, 43, 44, 45, 52, 53 and 54.

intended application; which involved relatively high apparent gravity fields in the first[53] and low gravity fields in the second[54]. The exceptionally low values of Reynolds Numbers achieved by Shih[54] resulted from tests conducted in motor oil, the high values by Cole[53] by a combination of multiple plate widths, high frequency and heated water. It should be noted that the large range shown for Ridjanovic[22] is deceptive and possibly misleading for two reasons, 1) some guessing had to be done and 2) the study was on the influence of plate aspect ratio so that a large range in "D" is present in the data.

The estimation of the range of non-dimensional frequency ($\omega D^2/\nu$) achieved in each experiment is in some cases slightly less speculative. As before, at least approximate ranges of this frequency parameter were derived, and the resulting ranges of non-dimensional amplitude and frequency are shown in Figure 2, where the boxes defining the ranges are labelled with the appropriate reference number.

Figure 2, which is a comparison of non-dimensional amplitudes and frequencies, is the sort of simulation plot implied by alternate non-dimensional variable Forms 3, 4 and 7 of Table II, while Figure I would result from Forms 1, 2 and 5.

When the situation is viewed from an amplitude/frequency perspective (Figure 2) the various sets of experimental data appear to involve much less overlap than would be concluded from Figure 1. In both Figures the parameter range for the basic work of Keulegan[26] has been represented more precisely than the others because all data was conveniently available. As may be noted Keulegan[26] achieved a wide range of frequency by varying plate width but a very narrow range of amplitude at each frequency. Something like the same situation might have been obtained had a more detailed dissection of the other experiments been carried out in preparing the Figures. The results of Cole[53] show a definite frequency dependence. This suggested that the unexplained difference between results of Keulegan[26] and Paape[43] for roughly the same Reynolds Number (Figure I) may be a frequency effect since the frequency range of Paape[43] is roughly between that of Keulegan[26] and Cole[53]. The results of Woolam[45], Brown[42], and Stephens[52] indicate a relatively small frequency effect in their domain of A/D; Henry[44] obtained a somewhat more pronounced effect of frequency.

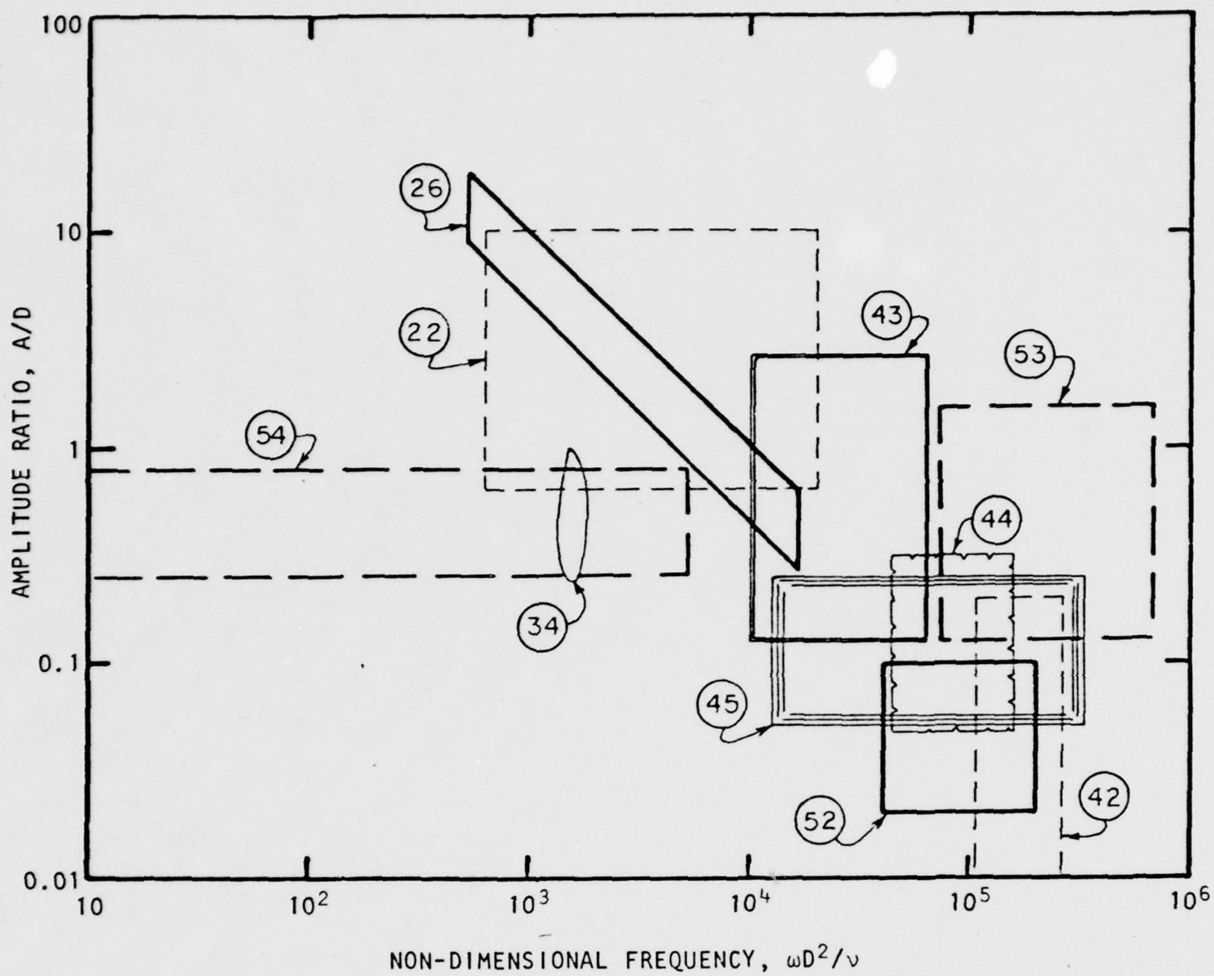

FIGURE 2 APPROXIMATE RANGES OF NON-DIMENSIONAL AMPLITUDE AND FREQUENCY ACHIEVED IN PLATE EXPERIMENTS, REFERENCES 22, 26, 34, 42, 43, 44, 45, 52, 53 and 54.

It is generally clear from the literature cited that the averaged oscillatory drag coefficients for flat plates are not wildly sensitive to unsteady Reynolds Number. Large changes are brought about only by very large changes in Reynolds Number. A comparison of Figures 1 and 2 suggests that correlations of the results of various investigators may have been complicated by the use of the (mixed) Reynolds parameter in addition to the edge effects noted by Tseng[41].

While considering something akin to simulation, it is interesting to note a range of non-dimensional amplitude and frequency appropriate to the simulation of ship bilge keels. Assuming that "D" of practical interest might be within 0.3 to 2 m., a range of frequency covering both small and large ships might be $0.2 < \omega < 1.5$. Combining these ranges, the range of frequency parameter of interest becomes approximately

$$2 \times 10^4 < \omega D^2/\nu < 5\times10^6$$

It is not difficult to imagine a range of A/D between 0.1 and 10 in this application. Comparing these ranges with Figure 2 it is evident that a portion of the range of interest corresponds to the ranges which appear to have been achieved by Cole[53] and Paape[43].

PLATE DATA FROM AN AMPLITUDE/FREQUENCY PERSPECTIVE

Though there are minor deviations in the literature surveyed, the essential ideas about the derivation of average drag and mass coefficients are contained in Keulegan and Carpenter[26]. Assuming a cosinusoidal variation of U, Equation 5; in accordance with the conventions of Keulegan[26] (the negative of Equations 1):

$$\dot{X}(t) = U = -U_m \cos\omega t$$

after substitution, Equation 5 becomes:

$$F = \tfrac{1}{4}\,\rho\pi D^2 L\omega C_m U_m \sin\omega t - \tfrac{1}{2}\rho D L C_d U_m^2 |\cos\omega t| \cos\omega t \qquad (7)$$

(where F stands for total force on a plate of length L).

In order to cope with the second term of Equation 7 in a practical way it is assumed that the product, $|\cos\omega t|\cos\omega t$, may be replaced by the first non-zero term in a Fourier cosine series; That is:

$$|\cos\omega t|\cos\omega t \simeq \frac{8}{3\pi}\cos\omega t$$

Whereupon it is clear that Equation 7 may be expressed in the form:

$$F \simeq P_1 \sin\omega t + Q_1 \cos\omega t \qquad (8)$$

where:

$$P_1 = \tfrac{1}{4}\rho\pi D^2 L\omega U_m\, C_m$$

$$Q_1 = -\frac{4}{3\pi}\,\rho D L U_m^2\, C_d \qquad (9)$$

In the case that P_1 and Q_1 are evaluated from harmonic analyses of data; that is,

$$P_1 = \frac{1}{\pi}\int^{2\pi} F \sin\omega t\, d(\omega t)$$

$$Q_1 = \frac{1}{\pi}\int_0^{2\pi} F \cos\omega t\, d(\omega t) \qquad (10)$$

There results Keulegan's[26] estimator for averaged mass and drag coefficients:

$$\hat{C}_m = \frac{P_1}{\frac{1}{4}\pi\rho\omega D^2 L U_m}$$

$$\hat{C}_d = \frac{-3\pi Q_1/8}{\frac{1}{2}\rho D L U_m^2} \qquad (11)$$

The denominators in Equations 11 are those conventionally used throughout the literature. The symbol "P_1" is the amplitude of force in-phase with acceleration; the numerator of the drag coefficient estimator is an effective amplitude of force which is in-phase with velocity. The Fourier analysis method of Keulegan[26] was apparently used by Cole[53] and Paape[43]. In order to reduce labor Shih[54] and Woolam[45] (in his forced oscillation experiments) used simpler measures of the effective drag amplitude. All the remaining experiments under consideration were decay experiments and an effective drag amplitude is implicit in the analysis and data reduction procedure.

In order to attempt a correlation of data from an alternate point of view it was thought that it might be instructive to derive the effective harmonic coefficients P_1, Q_1, from the given drag and mass coefficient data. Keulegan [26] gives the Fourier coefficients derived from the basic data, but this reference is the <u>only</u> one which does. Essentially, correlations of available data from an amplitude/frequency perspective are not possible unless something can be done with derived coefficients of the form of Equations 11. Accordingly, non-dimensionalizing Equation 8 in the alternate way previously mentioned.

$$\frac{F/L}{\rho\nu^2/D} = \underline{\underline{P}} \sin\omega t + \underline{\underline{Q}} \cos\omega t \qquad (12)$$

where:

$$\underline{\underline{P}} = P_1 D/\rho\nu^2 L = \tfrac{1}{4}\pi(\omega D^2/\nu)^2 (A/D)\ \hat{C}_m$$

$$\underline{\underline{Q}} = Q_1 D/\rho\nu^2 L = -\frac{4}{3\pi}(\omega D^2/\nu)^2\ (A/D)^2 \hat{C}_d \qquad (13)$$

In this form the $\underline{\underline{P}}$ and $\underline{\underline{Q}}$ coefficients represent an <u>estimate</u> of the non-dimensional fundamental Fourier coefficients of the force per unit length of the plate, given experimentally derived mass and drag coefficients and

non-dimensional frequency and amplitude. As previously noted, this form of non-dimensionalization involves no prior assumption of the form of the relationship between dependent and independent variables.

SOME CORRELATIONS OF TWO-DIMENSIONAL PLATE DATA

There are in the cited references on plates, five which deal with two-dimensional experiments (Keulegan[26], McNown[34], Paape[43], Martin[21], Cole[53] and Shih[54]). One experiment of Ridjanovic[22] appears to involve a plate of such large length to width ratio that the results appear likely to be nearly two-dimensional according to comparisons with Martin's results[21]. The remainder of the references noted in Figures 1 and 2 (Ridjanovic[22], Brown[42], Henry[44], Woolam[45], Stephens[52]) involve three dimensional experiments involving rectangular plates of various aspect ratios as well as circular disks.

Considering the two dimensional experiments, there are only two where averaged mass coefficients as such were derived or presented. These are Keulegan[26] and Paape[43]. Unfortunately, no detail is given by Paape[43] as to what experimental oscillation frequencies correspond to each data point. Essentially, analysis of any of the data of Paape from an amplitude/frequency perspective was completely frustrated by this and no further use could be made of the reference. This means that only one source was available for estimates of $\underline{\underline{P}}$, Equation 13. Figure 3 contains all the values of $\underline{\underline{P}}$ derivable from Keulegan[26]. For each of the eight values of non-dimensional frequency which were achieved, values of $\underline{\underline{P}}$ are plotted as functions of amplitude ratio, A/D. (Numerical values of the frequencies are noted).

The first thing immediately apparent from Figure 3 is that the Fourier components in-phase with acceleration are both frequency and amplitude dependent in an apparently systematic way. The second is that the trend of results for two frequencies (4900 and 3655) do not much resemble those for higher and lower frequencies. These particular data help produce the pronounced dip in the average inertia coefficient for plates given by Keulegan[26]. The range of amplitude ratios achieved in Reference 26 appear inadequate to define the implied function of two variables.

The prospects for correlation of two dimensional results from various

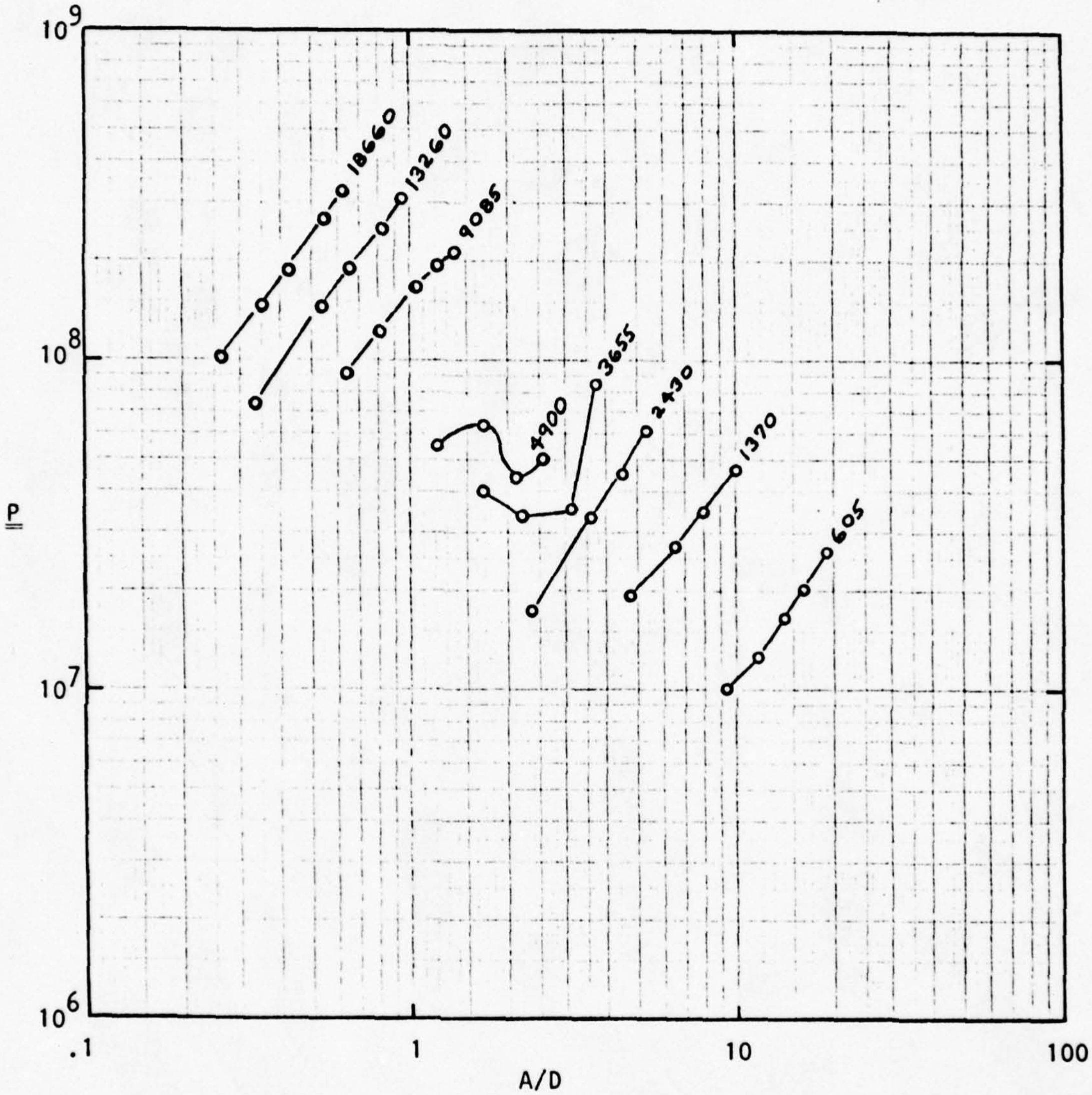

FIGURE 3 ESTIMATED FOURIER FORCE COMPONENTS IN-PHASE WITH ACCELERATION, AS FUNCTIONS OF AMPLITUDE RATIO FOR VARIOUS NON-DIMENSIONAL FREQUENCIES. DATA FROM REFERENCE 26.

experiments are much better when the component in-phase with velocity (the "drag" coefficient) is involved, since this is the only thing most investigators have had an interest in. Because of the differences in test techniques, the data falls naturally into two categories: "Drag" components as a function of amplitude for constant values of frequency; and "drag" components as a function of frequency for constant values of amplitude. Figure 4 summarizes some results of the first category in non-dimensional form. Specifically, the negative of $\underline{Q}$ (Equation 13) is plotted as a function of A/D for various (noted) values of non-dimensional frequency. The open circles in Figure 4 are from the data of Keulegan[26] and correspond to the data for $\underline{P}$ in Figure 3. Because it was readily available, the drag coefficient data of Ridjanovic[22] for plate aspect ratio of 19.4 ($\omega D^2/\nu=811$) was converted to the present form and plotted for comparison. Unfortunately, the numerical data required for the present exercise are not all given by McNown[34]. From the data which was given it was possible to make a very rough estimate from this reference for two points at a non-dimensional frequency of 1744 and these are plotted in Figure 4. (In this connection it should be noted that it was necessary to evaluate amplitudes from theoretical expressions given in Reference 34, and that the amplitude of force in-phase with velocity had to be approximated as the instantaneous force at time of maximum velocity since time histories of a complete cycle of force were not given).

The odd behaviour of the Keulegan[26] data for frequencies of 4900 and 3655 shown in Figure 3 for $\underline{P}$ is not reflected in the data for $-\underline{Q}$, Figure 4; all the $\underline{Q}$ results from Keulegan[26] form an apparently systematic family with comparable trends regardless of frequency. The data of Ridjanovic[22] and McNown[34] appear to at least roughly fit into the pattern, though the trend with amplitude of the two points from McNown[34] (Frequency= 1744) does not agree with the trends from the other two sources.

From the viewpoint of dimensional analysis, the data comparisons in Figures 3 and 4 involve an ignored parameter, the plate thickness. This parameter appears to be about the only thing not documented by Keulegan[26]. From the photographs in that reference it appears that the width-thickness ratio for the largest plate tested (D=0.076m.) was in excess of 20 and that the edges were squared off rather than sharpened. If, as seems likely, the

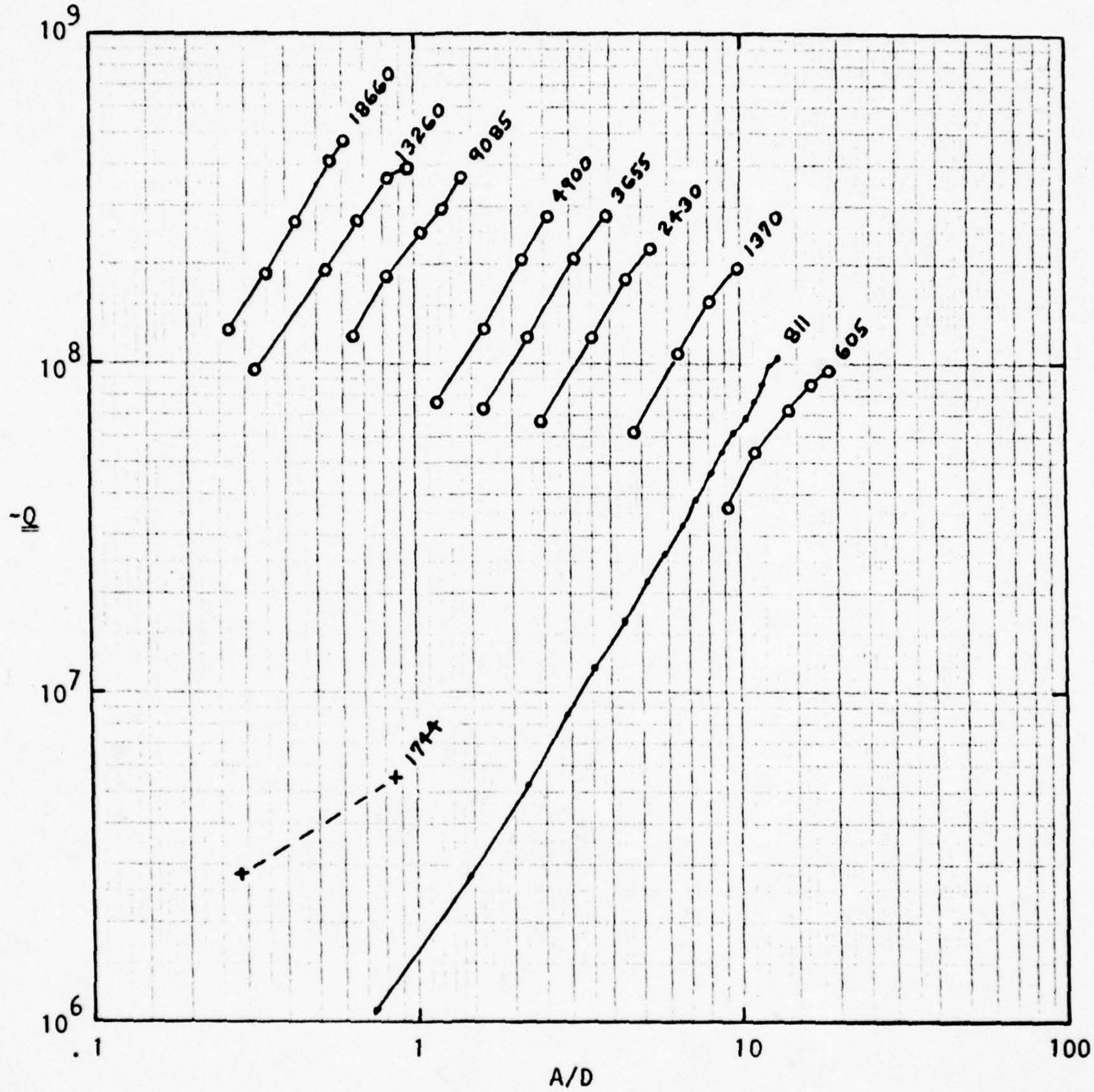

FIGURE 4 ESTIMATED FOURIER FORCE COMPONENTS IN PHASE WITH VELOCITY AS FUNCTIONS OF AMPLITUDE RATIO FOR VARIOUS NON-DIMENSIONAL FREQUENCIES (DATA FROM REFS. 26, 22, 34)

same thickness was retained down to the smallest plate (D=0.013m.) the width/thickness ratio would have been about 4. Translated into terms of non-dimensional frequency it appears likely that width/thickness ratios for the Keulegan[26] data ranged from above 20 for $\omega D^2/\nu$ = 18660 to about 4 for $\omega D^2/\nu$=605. The plates of both McNown[34] and Ridjanovic[22] were apparently squared off; that of McNown had width/thickness ratio of 8; that for the data of Ridjanovic shown in Figure 4 (frequency =811) had width/thickness ratio of 10. The data comparisions in Figure 4 are much too sparse for conclusions, but it seems likely from the comparison of the relative position of the data for frequency of 811 (width/thickness of 10) with the data for frequencies of 605 and 1370 (probable width/thickness ratios of 4 to 6) that the thickness effect may not be profound.

The two data sources involving "drag" determinations as functions of frequency for constant amplitude are Cole[53] and Shih[54].

The experimental data of Cole involved a splitter plane so that in converting this data to the present non-dimensional form it was necessary to assume in effect that the force on a half plate is half the force on a plate of double width without a splitter plate. Relative to steady state drag data (Hoerner[55*]) the assumption would be considered to result in over estimates of the order of 40%. Whether or not this is the case in unsteady flow is not documented. Width/thickness ratios for the plates tested by Cole[53] are not documented. However, since the half widths (D/2) ranged from 0.075m. to 0.30m., and since it is likely that commonly available plate stock was used in fabricating models, it seems fair to assume that effective width/thickness ratios were in excess of 10 at least. The data of Cole[53] has one feature not found in the other available sets of data. This is that, for two particular values of A/D, data was obtained for three plate widths, various frequencies and two values of kinematic viscosity. When the data is converted to the present non-dimensional form there occurs a considerable overlap of the non-dimensional frequency ranges achieved for each plate width. The particular data involved is shown in non-dimensional form in Figure 5. The two amplitude ratios were

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*55. Hoerner, S.F., "Fluid-Dynamic Drag", Published by the Author, Midland Park, N.J., 1965.

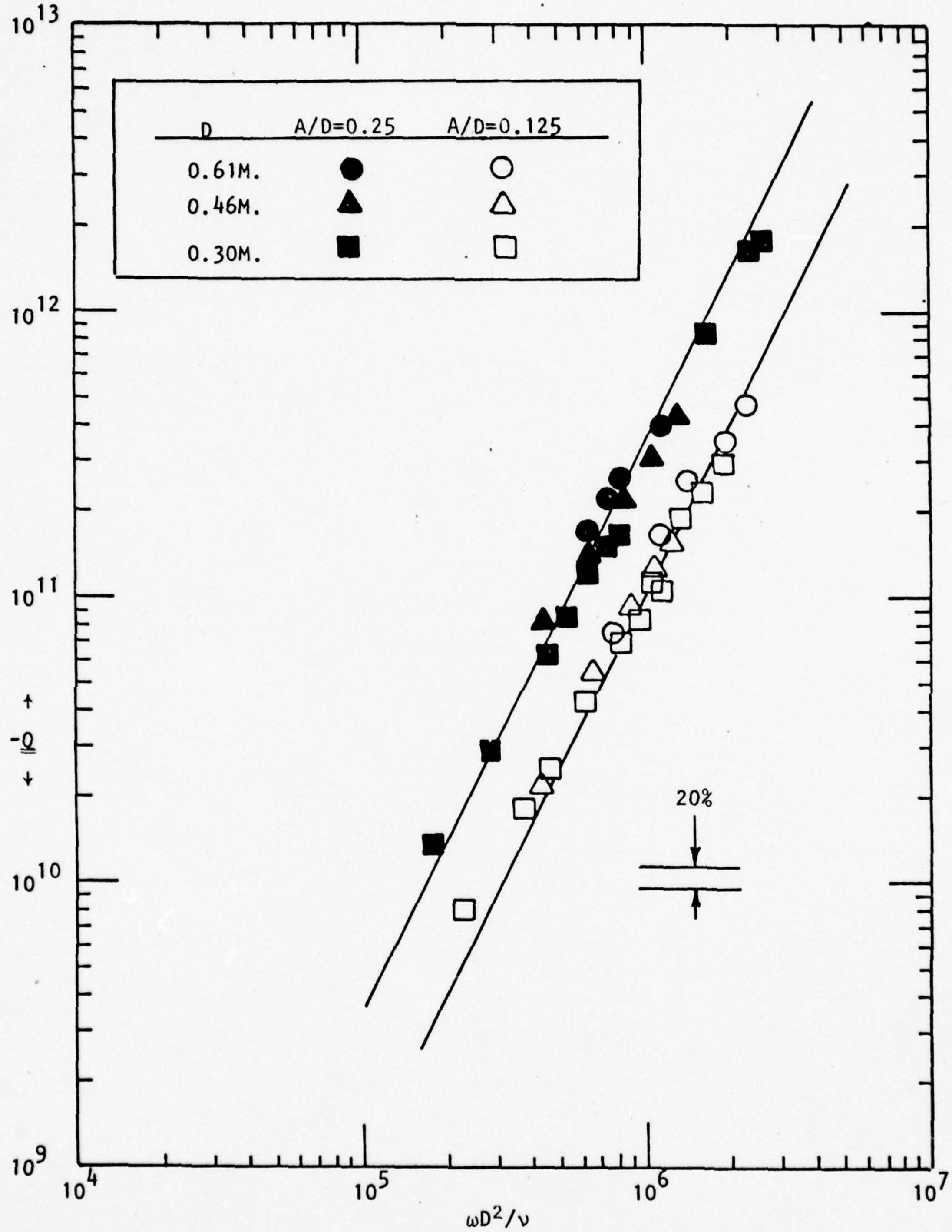

FIGURE 5 ESTIMATED FOURIER FORCE COMPONENTS IN PHASE WITH VELOCITY AS A FUNCTION OF FREQUENCY FOR PLATES OF THREE WIDTHS (REFERENCE 53)

0.25 (filled symbols) and 0.125 (open symbols). The range of frequency where the results overlap is from about 4×10^5 to 2×10^6. Within this range, data from the three different plate widths appear to compare within something like 20%. On average, there appears a very slight tendency for the component $-\underline{Q}$ to increase with plate size. However, since Cole[53] quotes experimental precision at no better than ±10% because of the necessary tare corrections to the data, ascribing the differences shown in Figure 5 to real effects of size or plate edge condition seems a doubtful proposition. In Figure 5, for both amplitude ratios, the three or four highest frequency points for plate width of 0.30m. were obtained in nearly boiling water, and, as may be seen, these points form a reasonable continuation of the data obtained in water at normal temperature.

The experiments of Shih[54] involved plates of two widths, which apparently had squared off edges and width/thickness ratios of 8 for the 0.0254m. wide plate and 4 for the 0.0127m. wide plate. The use of both motor oil and water for fluids resulted in a four decade range of non-dimensional frequency, but without overlap, so that direct comparisons of results in the two fluids is not possible. Though three different amplitude ratios were achieved, there is no case in which results were produced for the same amplitude ratio for plates of different widths. The data reduction methods utilized for the bulk of the data in Reference 54 were crude in relation to the methods implied by Equation 10. Essentially, the peak measured forces were assumed to be the effective drag component ($3\pi Q_1/8$), and spot checks using the methods implied by Equation 10 on two runs at a fairly high real frequency were made to confirm the adequacy of the approach. No correction of the data for the apparatus inertia tares was made, it being assumed that the force in-phase with velocity was dominant. No definite statement of estimated experimental accuracy was made. Considering the data reduction method, the lack of inertia tare correction, the degree of repeatibility of the force calibration curve, and the scatter of results in those instances where nearly repeat runs were made, suggests that experimental accuracy in this case was most unlikely to be better than ±15%.

Given all the caveats about the plate data just described (non-uniformity of width/thickness ratio, splitter plane in one instance, non-correction of inertia tares in another, etc., etc.) it would seem that not terribly good correlation should be anticipated. Nevertheless an attempt at a cross correlation of all the derived values of $\underline{Q}$ was made. The results of the

first stage in this attempt are shown in Figure 6. As shown in the figure, when correlations of all available data are considered, a 12 decade range is required for the estimated Fourier component in-phase with velocity ($\underline{Q}$), and a 7 decade range for non-dimensional frequency. The convention of the figure involves $\underline{Q}$ as a function of $\omega D^2/\nu$ for constant values of A/D. Twelve different values of A/D are involved (0.0625 through 20.) and the data shown were obtained in various ways. The convention of the figure is appropriate for direct plotting of the estimates obtained from Shih[54] and Cole[53]. The open symbols shown for $\omega D^2/\nu$ less than 10^4 are the tabulated data points of Shih[54] after re-non-dimensionalization (neglecting two points which Shih himself threw out). Three amplitude ratios are involved (0.75, 0.33 and 0.25), and the data clusters in two groups according to the test fluid (motor oil or water). At the upper right corner all the data obtained by Cole[53] are plotted in open symbols, crosses, or plus signs. Five amplitude ratios are involved (0.0625, 0.125, 0.25, 0.50, and 0.75) and the distinction between plate widths made in Figure 5 has been omitted in the results for A/D = 0.25 and 0.125. Since the data shown in Figure 4 as a function of A/D appeared systematic, this data was interpolated (and occasionally extrapolated slightly) at nine particular values of A/D (.25, .5, .75, 1., 2., 3., 5., 10. and 20.) and the resulting points plotted in Figure 6 are filled in. Thus, directly or indirectly, the two-dimensional data of five investigations (References 22, 26, 34, 53, 54) are included in the figure. Through the data for each value of A/D a mean line (to be later discussed) has been drawn, and the corresponding numerical value of A/D has been indicated above each line.

Two additional lines, neither supported by actual data, have been included in the figure. These were for the purpose of putting some perspective on a relatively unconventional correlation plot.

The line in the upper part of the figure (centerline convention) is a round-house estimate of where data for A/D=1000 might be. It is derived quite simply by assuming that the amplitude of force at this oscillation amplitude ratio is the same as the force on a two-dimensional plate in steady flow with velocity equal to U_m.

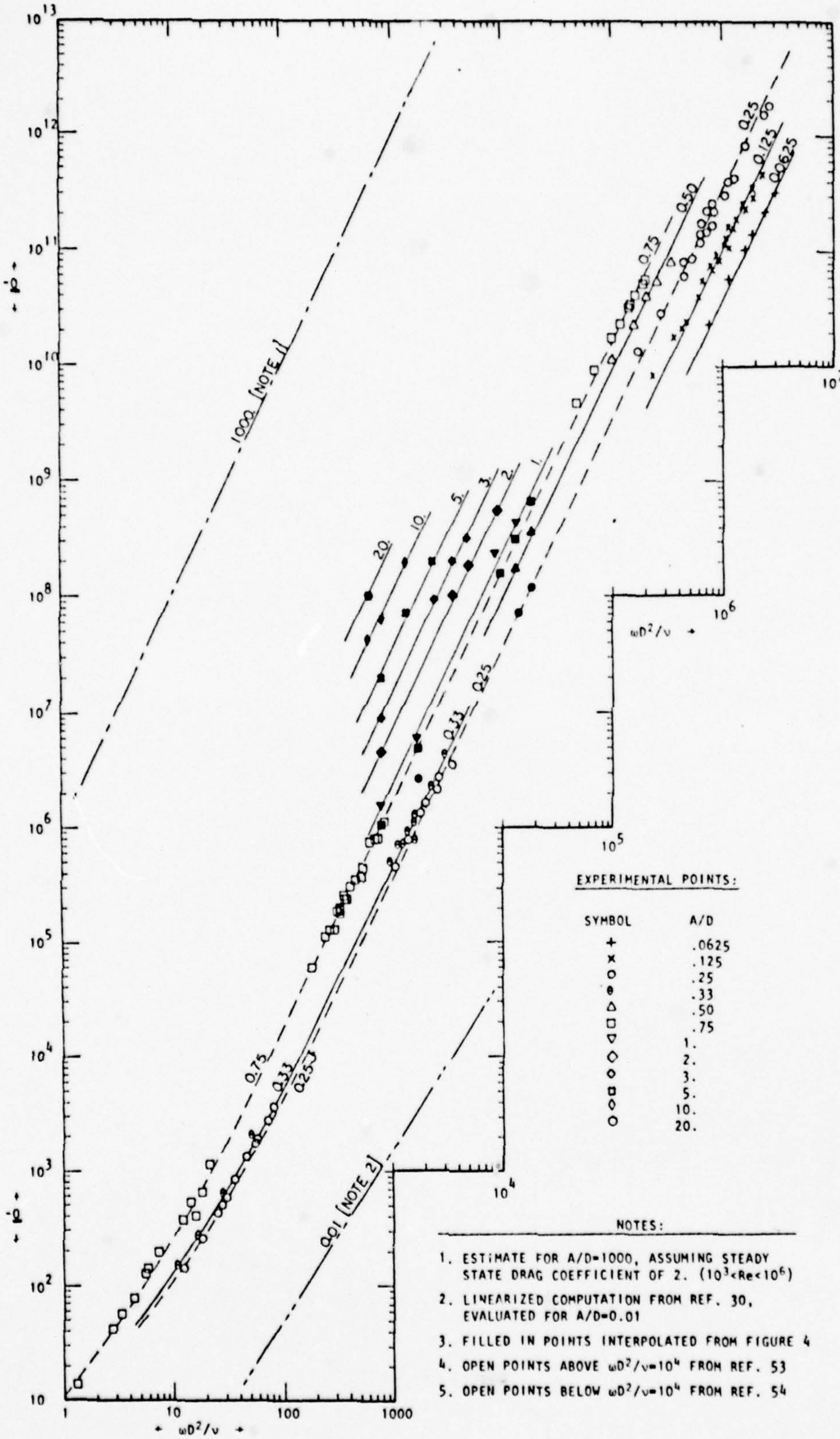

FIGURE 6 CORRELATION OF Q, $\omega D^2/\nu$, AND A/D DERIVED FROM RESULTS OF VARIOUS INVESTIGATORS

Assuming a drag coefficient of 2, the force per unit length would be

$$F/L \simeq \rho D U_m^2$$

Non-dimensionalizing in the present manner:

$$\frac{F/L}{\rho \nu^2/D} \simeq (Re)^2 = (\omega D^2/\nu)^2 (A/D)^2$$

and the line in Figure 6 is this expression evaluated for A/D=1000. The steady state drag coefficient assumption just made seems to be accepted for $10^3 < Re < 10^6$, and the extent of the line shown in Figure 6 covers this entire range.

It is perhaps in order to clarify this steady state assumption. In a practical sense, "steady state" is defined by the length of time required for experimental starting transients to die out and force to become constant. One way of looking at the relationship between steady state and oscillatory motion is to say that "steady state" is what happens when frequency is zero. This is not too productive since if frequency is zero, maximum velocity of sinusoidal motion is also zero and there is no way to generate a force. An alternate approach is to say that for practical purposes "steady state" may be approached for a relatively brief time during each half of a cycle of very large amplitude motion. To be specific, if the amplitude ratio (A/D) of the oscillatory motion is 1000, then, in the vicinity of maximum velocity, the actual velocity is within 0.1% of constant during a time that the plate travels about 100 widths, and within 1% of maximum velocity during a time that the plate travels about 280 widths. For plates the usually accepted figure for the distance traveled by the plate in the time required to shed a pair of vortices in a Karman vortex street is about 7D. Thus with A/D= 1000, about 14 vortex pairs would be shed while velocity is within 0.1% of constant and about 40 while velocity is within 1% of constant. These numbers are quite comparable with durations, lenths of travel and velocity accuracy which might be experienced in actual experiments. Judging by towing tank practice, essentially steady state drag ought to be achieved in shorter distances than just mentioned, and it seems reasonable to expect the peak forces for A/D of 1000 to resemble steady state. The reasons that this estimate of the fundamental Fourier component can only be considered

order of magnitude are that the force history according to the constant drag coefficient assumption is very non-sinusoidal, and that the constant drag coefficient assumption is incorrect when very low velocities are considered. (Hoerner[55] is full of examples showing that steady flow drag coefficients are not constants over large ranges of Reynolds Number).

Returning to Figure 6, the next item is the line at the lower right in the section convention. The line is labelled 0.01 and represents the linearized computation of Tuck[30] evaluated for A/D of 0.01. Vortex generation was explicitly excluded from this theoretical effort, and it seems likely that the theory may hold only for much smaller A/D ratios. Evaluation for A/D=0.01 was chosen merely to put the line into the field of the plot because what was desired was to display the general trend which is roughly parallel to $(\omega D^2/\nu)^{1.5}$.

A general inspection of the data points in Figure 6 suggested that above a frequency of 100 the variation with frequency was much the same as that of the steady state estimate (frequency squared), while below this frequency the trend of data resembled that of the linearized computation. Tuck[30] derived an asymptotic solution for the forces on a two dimensional plate as frequency (and amplitude) approach zero. After conversion of this solution to the present non-dimensional scheme there is obtained:

$$\underline{\underline{Q}} \rightarrow 8\pi(A/D)\ (\omega D^2/\nu)\ g(\omega D^2/\nu) \qquad (14)$$

where:

$$g(\alpha) = -\ \ell n(\alpha/4)/\{\pi^2 + \{\ell n(\alpha/4)\}^2\} \qquad (15)$$

(The same form may be obtained by some manipulations and small argument assumptions from Stokes' solution for the circular cylinder). The limit of the expression, Equation 14, as $(\omega D^2/\nu)$ approaches zero is zero. (Force should be zero for zero motion). Because of the log functions in $g(\alpha)$, the expression, Equation 14, begins to be very close to a constant times frequency for extremely small frequency.

The foregoing considerations suggested that a reasonable empirical model for $\underline{\underline{Q}}$ for constant A/D might be:

$$\hat{\underline{\underline{Q}}} = C_1\ (\omega D^2/\nu) + C_2(\omega D^2/\nu)^2 \qquad (16)$$

where C_1 and C_2 are functions of A/D.

Relative to an examination of gross behavior of the drag component as a function of frequency, there are two amplitude ratios of particular interest in Figure 6. These are A/D=0.75 and 0.25, since in these cases there are data points which cover nearly the entire experimentally determined range of $\underline{Q}$ and $\omega D^2/\nu$. Accordingly, Equation 16 was fitted to all available data (including cross faired results) for these amplitude ratios. Because of the large variation in magnitudes, the fitting method involved choosing C_1 and C_2 so as to minimize the mean squared percentage deviation of the data from the fitted line. The results of this procedure appear as dashed lines in Figure 6, and the numerical values of the coefficients and root mean square percentage deviation are shown in Table III. Considering the range and diversity of the data from the various sources, these fits are astoundingly good. Considering the results for A/D=0.75, the RMS percentage deviation of 13% is comparable to the probable precision of the basic data. Regardless of the data source there are few if any prominent deviations from the line. The general conclusion from an examination of the line for A/D=0.25 is similar except that the percentage deviation is higher because of the rather larger scatter of the grouped data of Cole[54] (see Figure 5) and because one of the cross-faired points is badly out of line. This particular point is at frequency 1744 ($\underline{Q}=2.5\times10^6$) and is the result of the writer's possibly faulty interpretation of the fragmentary results of McNown[34]. A similar fit was made to the data for A/D=0.33 and the result, shown as a solid line in Figure 6, is practically parallel to the line for A/D=0.25.

An examination of the fitted lines for A/D=0.75, 0.33 and 0.25 disclosed that, to a precision very much better than that of the data, the lines fitted in accordance with Equation 16 are essentially straight with a slope of 2 above a frequency in the vicinity of 100. This suggested that it would be appropriate to fit lines through the points for other A/D ratios with a simple square law since the range of frequency for other ratios is relatively restricted. The results of these fits produced the

TABLE III

Particulars of the Lines Fitted

To the Data in Figure 6

A/D	C_1	C_2	Root-Mean-Square Percentage Deviation
20	-	273.2	-
10	-	103.5	6.0
5	-	33.50	7.1
3	-	14.35	6.0
2	-	7.070	4.6
1	-	2.438	12.1
0.75	9.508	1.707	13.2
0.50	-	0.9343	13.0
0.33	8.947	0.4390	12.0
0.25	8.047	0.3614	25.0
0.125	-	0.1052	16.0
0.0625	-	0.03763	8.3

lines through the remainder of the data in Figure 6. The numerical values of the square law fits for each A/D ratio are given in Table III along with RMS percentage deviations. (The dashes in the column of Table III for C_1 indicate that C_1 in Equation 16 was assumed identically equal to zero). The resulting RMS percentage deviations are in line with the magnitude of data scatter.

On The whole, the data shown in Figure 6 appear as a systematically related family. The empirical adherance of the results for A/D of 0.75 and 0.25 to an uncomplicated trend, regardless of source, is the thing expected for geometrically similar experiments. The result suggests again that plate width to thickness ratios may not be wildly important in a relative sense, though the data is insufficient to be certain. The fact that the data of Cole[53] (with splitter plate) falls in line with the rest may simply mean that for small A/D ratios the boundary layer on the splitter plane has no time to develop, and that there is no time to develop a vortex street, and thus that the influence of the splitter plane was nil. (Good correspondence for higher A/D ratios might well not be expected). If the above surmises are correct, the conclusion of Shih[53] (that the unsteady drag coefficient, for a given amplitude ratio, is independent of unsteady Reynolds Number above about 200) is extended to Reynolds Numbers far beyond the range of data in Reference 54; in fact into the range appropriate to ship bilge keels. (Unsteady Reynolds Number is the product of A/D and $\omega D^2/\nu$; for constant A/D, a constant drag coefficient implies a frequency squared variation in force). In terms of non-dimensional frequency, the data available suggests that, above $\omega D^2/\nu$ of approximately 100, the drag component of force is proportional to frequency squared. Judging by the trend of the position on the fitted lines where curvature seems to cease, one might expect this lower limit to be at a higher frequency for very small A/D ratios, but there is no way to estimate how much higher from the data available.

From a practical point of view, non-dimensional frequencies much less than 100 are very difficult to achieve in water unless the plate and/or frequency are miniscule. For instance in the context of ship model rolling, a model with 0.01m. bilge keels might, upon occasion, experience motion frequencies corresponding to a non-dimensional plate oscillation frequency of 500, but it is hard to imagine much lower frequencies of practical interest.

If data for non-dimensional frequencies less than 100 are disregarded, the previous discussion implies that division of $\underline{\underline{Q}}$ by $(\omega D^2/\nu)^2$ should collapse all the remaining data into a single function of A/D. Since one non-dimensional variable is divided by another, the result is a new, mixed dependent variable; from Equation 13:

$$\underline{\underline{Q}}/(\omega D^2/\nu)^2 = Q_1/\rho\omega^2 D^3 L$$

$$\underline{\underline{P}}/(\omega D^2/\nu)^2 = P_1/\rho\omega^2 D^3 L \qquad (17)$$

Figure 7 indicates the results of carrying out such an operation on all the data shown in Figure 4, all the data of Cole[53], and the data of Shih[54] for which $\omega D^2/\nu$ was greater than 100. The results shown as open circles in Figure 7 are those from Figure 4 (References 22, 26, 34). All these data collapse well excepting for one point (A/D=0.29) which is one of the estimates made by the writer from McNown's paper[34]. For subsequent operations this point was disregarded. Data of Cole and Shih are shown as diamonds in the figure. The small cross indicates the mean of all data for the particular amplitude ratio and the vertical extent of the diamond includes the extremes of the data scatter. These data fit in very well with the collapsed data from Figure 4. Excepting the odd point derived from McNown[34] and including all the individual data points which lie within the diamond symbols, Figure 7 involves the collapse of 151 individual data points from five experiments.

It is clear that the collapsed data in Figure 7 lie along a roughly straight line on the log-log plot, and it was accordingly of interest to make a least RMS percentage error fit of a simple power law to the data. The result was the solid straight line shown. The best fit of such a law was found to be:

$$-Q_1/\rho\omega^2 D^3 L \simeq 2.66(A/D)^{1.54}$$

With a root mean square percentage deviation of 14%. Nearly as good a fit was obtained with an exponent of 3/2; in this case the RMS percentage deviation was 16%, the line is shown dashed in the figure, and the resulting formula was:

$$-Q_1/\rho\omega^2 D^3 L \simeq 2.63(A/D)^{1.50}$$

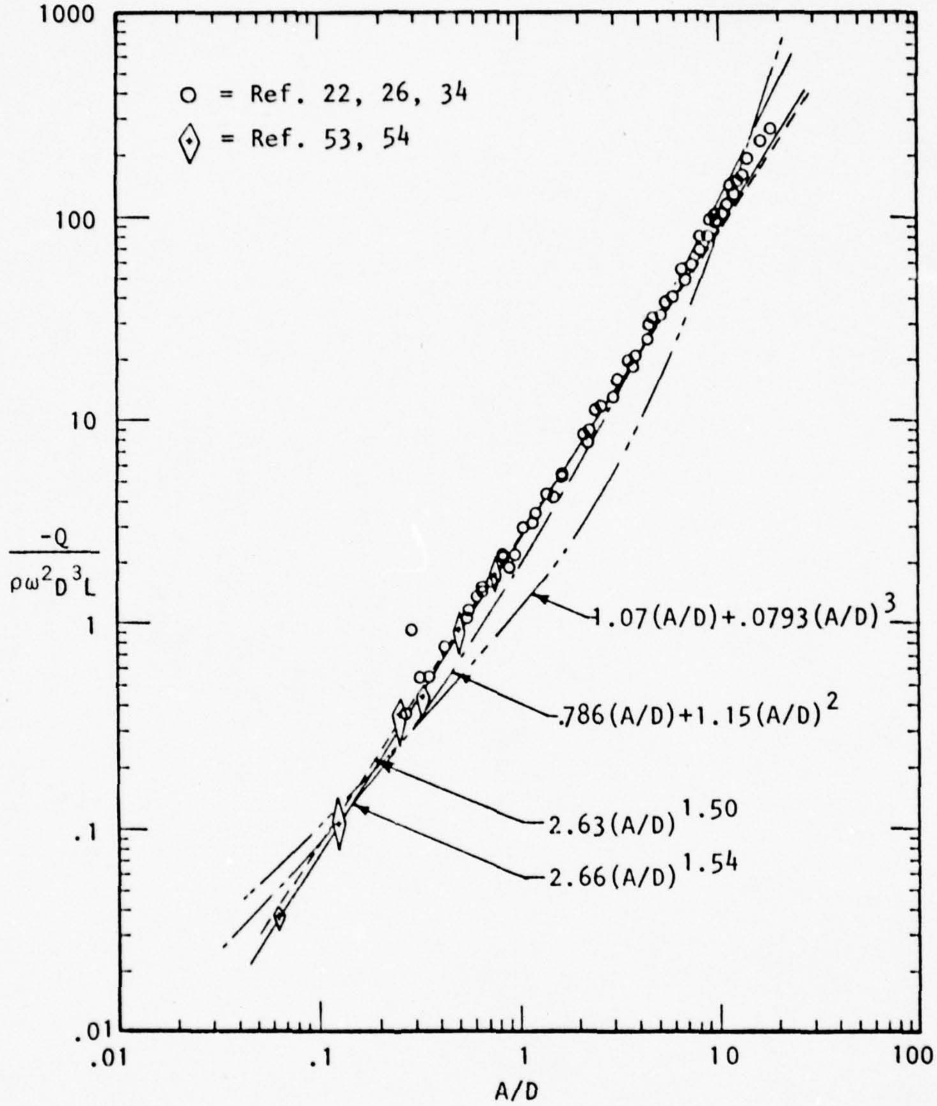

FIGURE 7 $\{-Q/\rho\omega^2 D^3 L\}$ vs A/D for $\omega D^2/\nu > 100$.
DATA FROM REFERENCES 22, 26, 34, 53, 54.

It is clear by inspection of the Figure that the data for A/D greater than about 8 trends slightly away from the fitted lines. Accordingly, it was of interest to see if polynominal models in A/D would fit the collapsed data. A fifth order polynominal was found to fit the data with a 17% RMS percentage deviation, but since some coefficients were negative, the resulting fitted equation produced values of Q_1 with opposite sign for very large A/D. The round-house estimates for A/D=1000 discussed in conjunction with Figure 6 imply that the function should continue roughly along the trend shown towards a value of $-Q_1/\rho\omega^2D^3L$ near 10^6 for amplitude ratios a decade higher than could be shown in Figure 7. The fifth order polynominal was therefore considered an over-fit and thus un-realistic. Much the same results were obtained with three term polynominals, and therefore the only polynominal fits achieved which were considered grossly realistic were two term:

$$-Q_1/\rho\omega^2D^3L \rightarrow E_1(A/D)+E_2(A/D)^2$$

$$\text{or:} \quad F_1(A/D)+F_3(A/D)^3$$

Numerical values for the fits of linear-plus-quadratic and linear-plus-cubic models are shown in Figure 7 in the labels for the resulting fitted lines. Root mean square deviation of the linear-plus-quadratic was 28%, that for the linear-plus-cubic was nearly 80%. It is clear that the only way to produce a convincing polynominal fit to the data in Figure 7 would be to disregard much of the range of A/D since two term polynominals can be made to resemble straight lines if the range of argument is sufficiently restricted.

The implication from this analysis that the "drag" component amplitude of force varies as $(A^{1.5+})$ apparently has one precedent; Tanaka's empirical formula[47] for work done by bilge keels implies that the amplitude of force varies as (plate velocity amplitude)$^{1.6}$. Thus Tanaka would perhaps have normalized forces with $(\omega^{1.6}\,A^{1.6})$ rather than $(\omega^2A^{1.5})$ as is implied in the present analysis. The real frequency range covered by Tanaka spanned less than an octave for each plate, and the corresponding non-dimensional frequencies for all three of his plates appear to have spanned a little more than a decade, a much smaller range than shown in Figure 6. Under these circumstances it is possible that the difference between $\omega^{1.6}A^{1.6}$ and

$\omega^2 A^{1.6}$ might be difficult to detect.

The "drag" component normalization implied by the usual treatment is $(\omega A)^2$ or U_m^2. Within the present non-dimensional scheme, the non-dimensional velocity amplitude is:

$$(A/D)(\omega D^2/\nu) = U_m D/\nu$$

$$= \text{The Unsteady Reynolds Number}$$

The necessary use of logarithmic scales in the present work carries with it the possibility that the adequacy of collapse of data is exaggerated by this choice of scale. To make sure that the correlation shown in Figure 7 was some sort of improvement over the conventional treatment, the values of $\underline{Q}$ for all the data represented in Figure 7 were plotted against the corresponding values of $U_m D/\nu$. The result (not shown here) was a multi-decade log-log chart similar to Figure 6. The gross trend of data plotted in this way was clearly according to (Reynolds Number)^2 as implied by the usual drag coefficient treatment, but the extreme scatter of existing data about a mean $(Re)^2$ line was ± 300% rather than the ± 30% or so in Figure 7.

If the "drag" and "mass" components of force are related it seemed possible that the data for force components in-phase with acceleration should also collapse under the normalization implied by Equation 17. Figure 8 indicates the results of applying this transformation to the data of Keulegan[26], Figure 3. There is clearly a significant collapse of data relative to that shown in Figure 3. The scattered points in the vicinity of A/D=2.5 arise from the data for $\omega D^2/\nu$=4900 and 3655 which were noted as having an odd trend in the discussion of Figure 3. There appear to be two branches to the collapsed curve of $P_I/\rho\omega^2 D^3 L$, Figure 8, and each branch appears to roughly follow the same power law which seems to best fit the "drag" components, Figure 7. Very approximate 3/2 power law fits to each branch are shown in Figure 8.

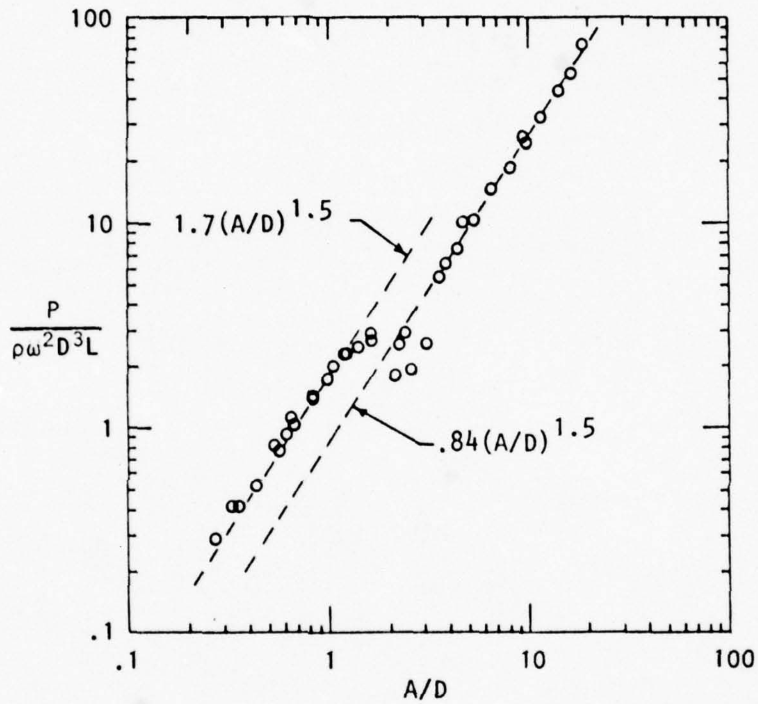

FIGURE 8 $\{P/\rho\omega^2 D^3 L\}$ vs A/D for $\omega D^2/\nu > 100$
DATA FROM REFERENCE 26.

CORRELATIONS OF DATA FROM THREE-DIMENSIONAL PLATE EXPERIMENTS

Up to this point all data considered has been essentially two dimensional. As noted earlier, there were five references involving experiments with rectangular and circular plates, and it was of interest to compare these data from an amplitude/frequency perspective where possible. The experiments of Ridjanovic[22] covered plate aspect ratios down to unity (the square plate). Square plates were also used in the experiments of Brown[42], Henry[44], and Woolam[45], so that in principle, correlations of four experiments were possible. The square plate results of Brown were found to be too compressed for present purposes and this reference was omitted from consideration in the square plate correlations. However, Brown gives sufficient faired data for the case of the circular disk so that a comparison with the circular disk results of Stephens[52] is possible.

Ridjanovic[22] tabulates results for a plate 0.1m. square which had a width/thickness ratio of 64. Because of the test technique, only one non-dimensional frequency is involved ($\omega D^2/\nu = 33000$), but amplitude ratios ranged from 2.1 to 0.11. The tests were carried out in water.

Henry[44] utilized an 0.12m. plate with sharpened edges and tested in water. Effective width to edge thickness ratios were undoubtedly very high. Non-dimensional frequencies were varied from 48000 to 158000 by varying the spring constants in a decaying oscillation apparatus. Unfortunately for present purposes, the analysis of data was confined to the portion of the decay curve where the slope implied linearity in some sense, and amplitude ratios corresponging to the numerical results were not given. However, a reasonable estimate for the range of amplitude ratios corresponding to the data is $0.03<A/D<0.1$.

Woolam's experiments[45] are extensively documented, and involve both free and forced oscillation experiments in air on three plates (D=0.25m, 0.50m, and 1.01m.) Width to thickness ratios on the basic tests were constant at 62.5. In some special tests the plate edges were effectively sharpened; the resulting width to edge thickness ratios were approximately

1500. Both forced oscillation and free decay type experiments were carried out. Non-dimensional frequencies spanned the range from 17600 to 895000.

To the extent possible, the data conversion implied by Equation 13 was exercised upon the drag coefficient data presented in the last three references. Plots similar to Figure 3 of $\underline{Q}$ vs A/D for various values of frequency were made, as were plots of $\underline{Q}$ vs $\omega D^2/\nu$ for constant amplitude ratio similar to Figure 6. The trends with frequency in the latter plot strongly suggested that the square plate data varied as frequency squared for constant A/D, so that it seemed reasonable to try the same normalization as had been used to produce Figure 7 for the two dimensional plate data. Noting that "L" in Equation 17 is the same as "D" in this case, the data normalization involves plotting $-Q_1/\rho\omega^2 D^4$ on a base of A/D.

The results of this operation are presented in Figure 9. The results of Ridjanovic[22] are shown as square symbols. No specific points could be developed for Henry's data[44], but a reasonable estimate of a range wherein his data should lie is shown by the dot shading. There are essentially three types of symbols utilized for Woolam's data. A diamond with a small cross indicates the results of forced oscillation experiments at five amplitude ratios (0.05, 0.075, 0.10, 0.15 and 0.20). The small cross indicates the mean of the data, the vertical extent of the diamond indicates the extreme spread. Within each diamond non-dimensional frequency varies by a factor of 4 to 8. The data symbolized by circles were developed by reading Woolam's faired drag coefficient curves at fairly closely spaced values of A/D. The open circles pertain to results for square edged plates (width/thickness=62.5) and the filled circles pertain to the thin edge case. Two 3/2 power law lines are shown. The line with coefficient of 2.63 has been shown to be a reasonable fit to the two-dimensional data, Figure 7; while the line with coefficient of 1.25 is what appeared to be a reasonable fit to the square plate data for A/D greater than 0.05.

If the results for A/D of 0.1 and greater are considered, the square plate data collapses about as well as the two-dimensional data (Figure 7). For this range of A/D the gross variation of both square and two-dimensional plate data appears to be of the same form, the differences being an approximate constant factor of two in the values of $Q_1/\rho\omega^2 D^3 L$. However below A/D of 0.1

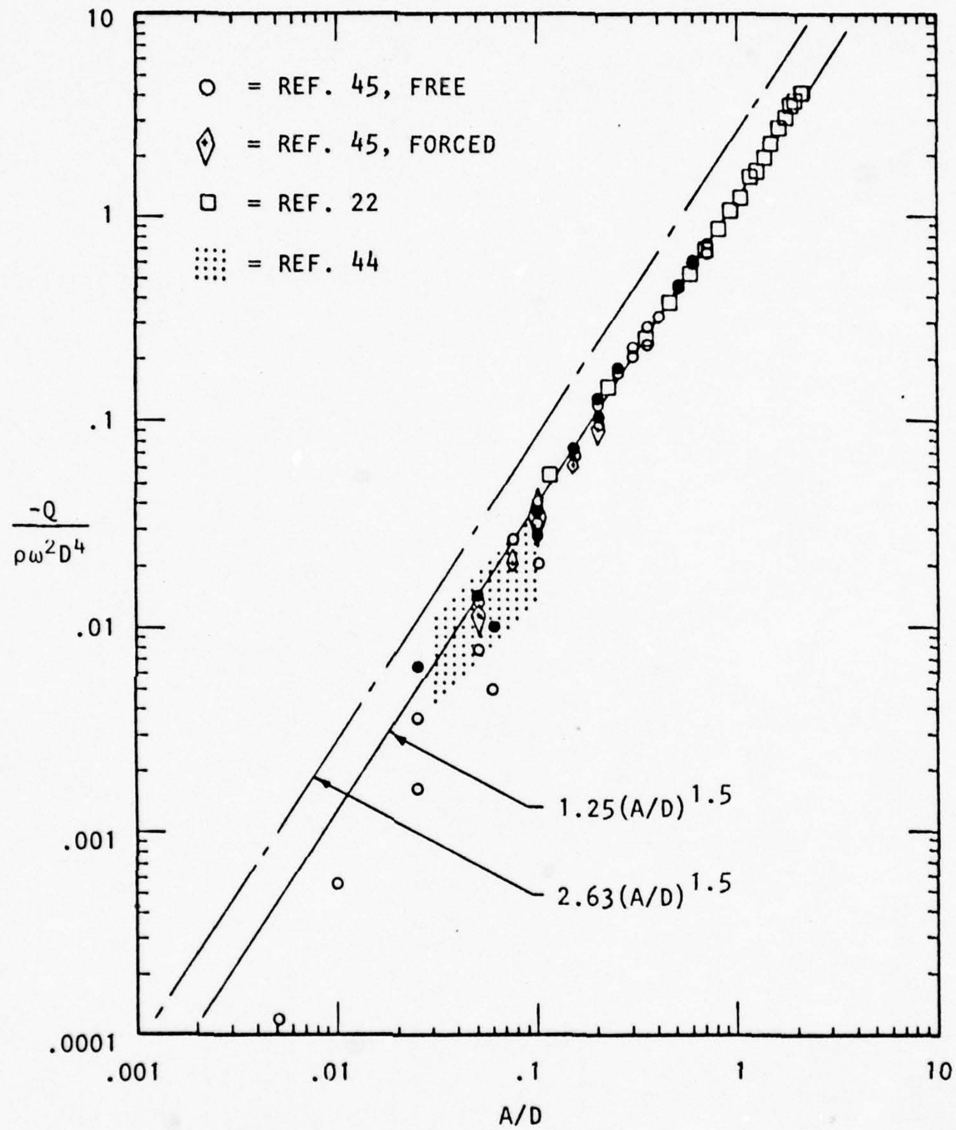

FIGURE 9 $\{-Q/\rho\omega^2 D^4\}$ vs A/D FOR SQUARE PLATES; NON-DIMENSIONAL FREQUENCIES VARY FROM 17600 TO 895000.

the correlation, the collapse of data, and the adherance to a 3/2 power law begin to unravel. The actual data shown for A/D less than 0.1 is due to Woolam who acknowledges that, due to the scatter of data from a large number of experiments, the accuracy of his faired result is most open to question for very low amplitude ratios and for his smallest plate. The writer's interpretation of the detailed results given by Woolam is that the range of possible answers obtainable for the data represented by the open circles furthest below the solid line in Figure 9 would lie between a factor of 2.0 and 0.5 of the spot shown. About the same margin could be applied to the point for the thin-edged plate shown at A/D=0.06. However, even with due consideration of the possible margins of experimental error, there appears to be a trend below A/D=0.05 to 0.1 for the square edge data (width/thickness = 62.5) to resemble a square law rather than the 3/2 power law. It appears that the thin edge case (filled circles) could continue on the 3/2 power law.

Turning to the two references from which data for the circular disk could be abstracted, Brown[42] conducted free decay experiments in water with a square edged 0.1m diameter disk of width/thickness ratio 32. Two values of non-dimensional frequency were achieved, 126000 and 300000; and results were read from faired curves for present purposes. Stephens[52] conducted free decay experiments in air at various pressures with a square edged 0.16m diameter disk of width/thickness ratio of 40. The experiment at a non-dimensional frequency of 38000 (atmospheric air pressure) was abstracted and the results converted to present form. It may be remarked that since Stephens gives only logarithmic decrements for a particular apparatus, conversion of this data required analytical development of some properties of the apparatus not given in the report[52].

The results from Brown[42] and Stephens[52] are shown in Figure 10 for the same non-dimensionalization used for the square plates. Scales and conventions of this Figure are the same as that of Figure 9, and the two 3/2 power law lines shown in Figure 9 are repeated to aid comparisons. From the point of view of the efficacy of the ($\rho\omega^2D^4$) normalization scheme for "drag" amplitudes, the collapse of disk data for high non-dimensional frequencies appears satisfactory. The trend of the result with A/D is clearly not according to a 3/2 power law. A mean line through the data

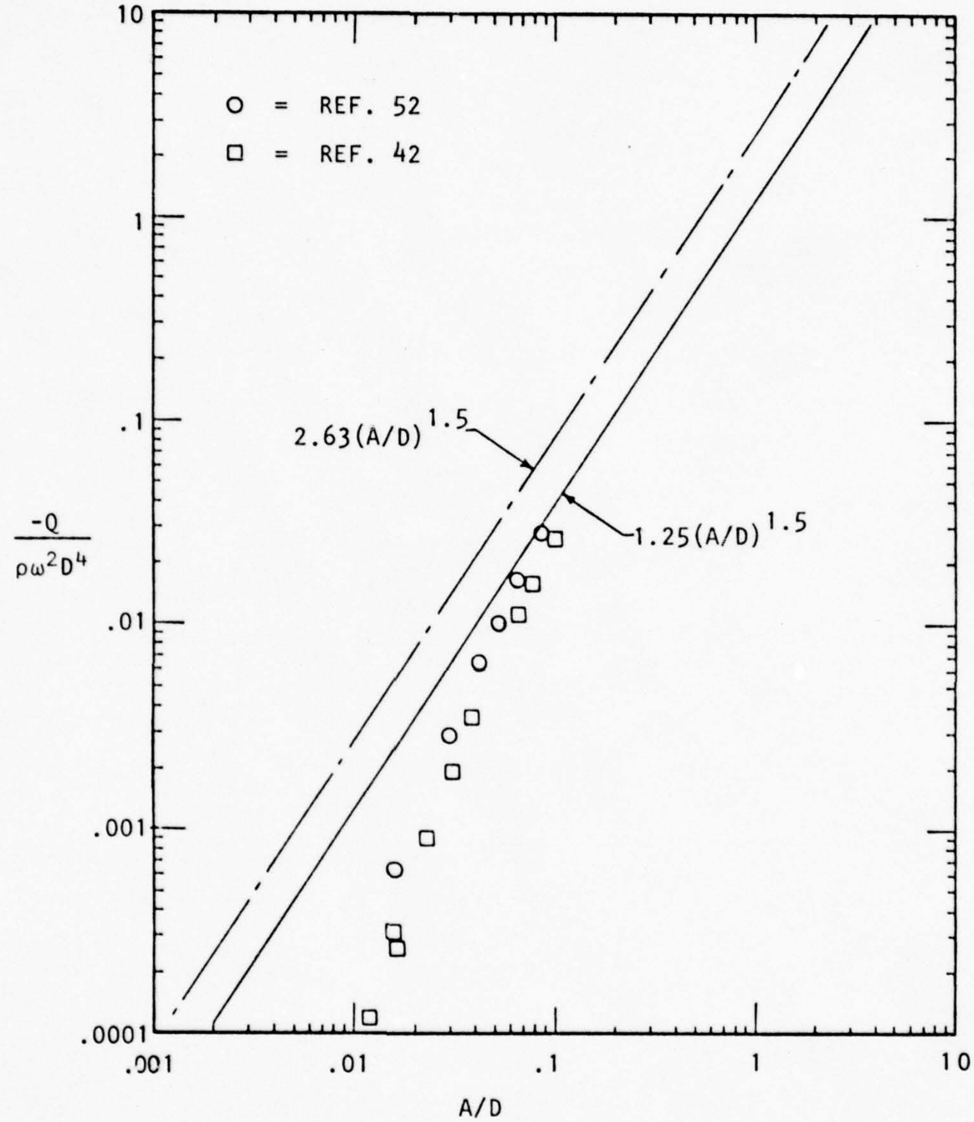

FIGURE 10 $\{-Q/\rho\omega^2D^4\}$ vs A/D FOR CIRCULAR DISKS: NON-DIMENSIONAL FREQUENCIES VARY FROM 38000 TO 300000

actually looks more like a constant times $(A/D)^{2.6}$.

In general, it appears from the results for plates and disks given in Figure 7, 9 and 10 that, for fixed amplitude ratio, the force in-phase with velocity (Q_1) on plates of a variety of aspect ratios varies approximately as the square of frequency so long as the non-dimensional frequency parameter $\omega D^2/\nu$ is greater than some critical value. Since $\omega D^2/\nu$ plays the same role in the amplitude/frequency system as the unsteady Reynolds Number plays in previous approaches to data correlation, there is an implication that the forces are independent of viscosity so long as the critical frequency parameter is exceeded. This is perhaps better expressed by the fact that the partial correlating modulus used to produce a collapse of data for given A/D ($\rho\omega^2 D^3 L$) does not involve viscosity. Reasonable collapse of data is achieved by this means for values of non-dimensional frequency up to the order of 10^6 which is well within the range which would be expected in the ship bilge keel situation.

Considering results for disks and for both square and two-dimensional plates, the available data suggests that, for fixed frequency, the force in-phase with velocity on plates of a variety of aspect ratios varies roughly with (A/D) to the 3/2 power above a critical value of A/D. The data suggests that for amplitude ratios in excess of 0.1, the 3/2 power variation holds regardless of width to edge thickness ratio greater than five or ten. The data also suggests that the break in the trend with A/D for fixed frequency comes at a critical A/D between 1/20 and 1/10 for width to thickness ratios in the range between 10 and 100. It is possible that very significantly sharper edges are required to drive the break point lower. The trend of force for amplitude ratios less than critical may be in the general vicinity of amplitude ratio squared. That data which exists is entirely too sparse and disconnected to inspire complete belief in an amplitude squared variation below the critical A/D.

Almost no data is found which corresponds to amplitude ratios less than about 0.01. Presumably linear theory (Tuck[30]) will begin to hold somewhere, but because of the generally reasonable collapse of data by squared frequency, it appears that all or virtually all data in hand was obtained at products of frequency and amplitude which are outside the range of validity of linear theory.

Above A/D ratios of 10 or 20 there is no data. Accordingly, no firm idea can be formed as to what happens as amplitude ratios grow significantly larger.

HIGHER HARMONIC COMPONENTS OF OSCILLATORY FORCES ON PLATES

The correlation and discussion of the last few sections have involved best estimates of the fundamental components of the unsteady forces on plates in oscillatory flow. Because the system appears to be non-linear, there is likely to be higher harmonic response as well. It was therefore also of interest to consider higher harmonic content in a way more or less consistent with the preceeding analysis of the fundamental components.

Of all the experimental references cited, Keulegan[26] is the only one in which systematic evaluation of higher harmonic content was carried out. Specifically, the forces were represented as an odd Fourier series:

$$F \simeq \sum_{n=1,3,5} \left(P_n \sin(n\omega t) + Q_n \cos(n\omega t) \right) \tag{18}$$

The integrations required were carried out to obtain the P_n and Q_n. (P_1 and Q_1 are the components previously treated). The resulting Q_n coefficients were modified to reflect the coefficients in the Fourier expansion of $|\cos\omega t|\cos\omega t$ which was required by the Morison Model, (Equations 5 and 7), and the resulting (Q_n') coefficients were tabulated along with the P_n. Reversing the modification process (obtaining Q_n from Q_n') is straightforward, and thus the original Fourier coefficients may be recovered to quite reasonable accuracy.

The first question of interest is the relative magnitudes of the higher harmonics. It is convenient to first consider the amplitudes of harmonics, which may be defined as:

$$|F_n| = \sqrt{P_n^2 + Q_n^2} \tag{19}$$

and these may be arbitrarily non-dimensionalized in accordance with the approach shown in Figure 7. The results of carrying out this operation on all data presented in Reference 26 are shown in Figure 11. In the figure

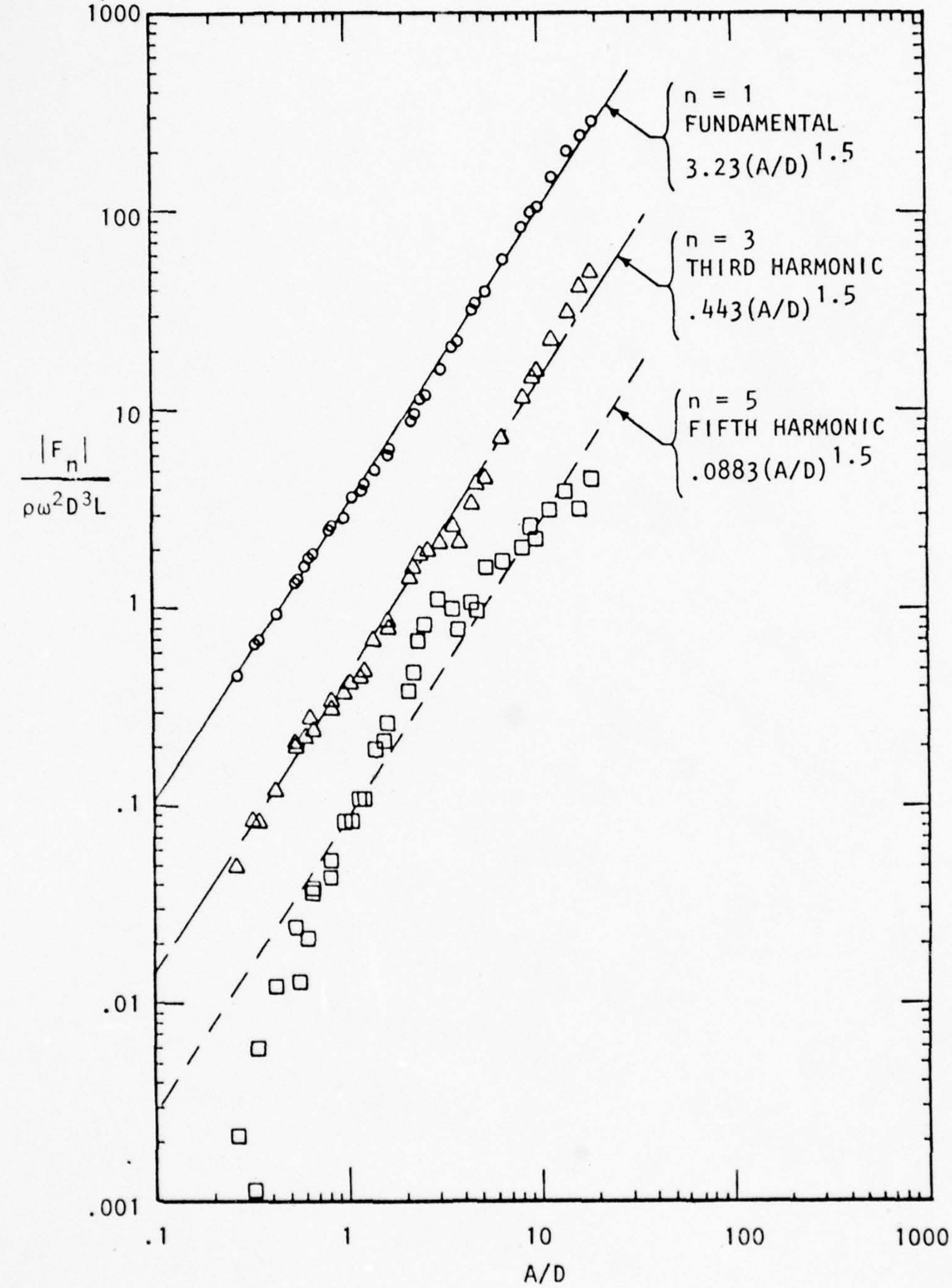

FIGURE 11 AMPLITUDES OF THE ODD HARMONICS OF THE FORCES ON PLATES OBSERVED IN REFERENCE 26 ($\omega D^2/\nu$ >600)

the non-dimensional amplitudes of first, third and fifth harmonics are indicated as functions of A/D. Through each of the three sets of data points in Figure 11 a 3/2 power law line has been drawn. These lines were fitted in a least root-mean-square percentage deviation sense. Numerical values of the coefficients are indicated in the labels.

Considering the fundamental component first, the fit of amplitudes to the 3/2 power law appears rather better than that shown in Figure 7 for the Q_1 or "drag" component, despite the odd trend shown in Figure 8 for the P_1 or "mass" component. The basic reason is that the non-dimensional P_1 components (Figure 8) are significantly smaller in magnitude than the Q_1 components (Figure 7).

The non-dimensional third harmonic component amplitudes shown in Figure 11 clearly follow the trend of the fundamental. On average the third harmonic component amplitudes are about 1/7 of the fundamental amplitude regardless of amplitude ratio.

The trend of fifth harmonic component amplitudes is clearly unlike that of the fundamental amplitudes. The majority of the fifth harmonic amplitudes shown are about 1/40 of the fundamental amplitude, though near A/D of 3 the ratio increases to the neighborhood of 1/20.

In the original Keulegan and Carpenter report (NBS Report 4821) there appears a passage which seems to have been deleted from the published version. This was to the effect that the magnitudes of the fifth harmonic component were of the order of the resolution of the basic force data. (Keulegan and Carpenter evidently thought so little of the fifth harmonic components that the coefficients were not tabulated in NBS Report 4821). The recording methods used by Keulegan[26] involved an oscillograph, and it may reasonably be assumed from the thoroughness of the study that attention was paid to filling up the linear range of the oscillograph with observation. However, even with this precaution, the basic resolution of force could not have been much better than 1/40 of the amplitude of the fundamental component. This means that the determination of fifth harmonic amplitudes of force may have been subject to considerable error, and under the circumstances, errors, systematic or random, of a factor of 2 or 3 can easily be imagined. As would be expected, the results in Figure 11 indicate rather clearly that scatter increases with the order of the harmonic.

Apart from oconcluding with Keulegan[26] that the magnitude of fifth harmonic of force is consistently very small, about the only other thing which might be observed is that a great deal more data would be required to define the behavior of this component satisfactorily.

As a preliminary to a further analysis of the third harmonic components, it was of interest to examine the relationship between the fundamental components P_1 and Q_1 in terms of phase. Defining ε_1 as the phase lead of the fundamental amplitude of force, $|F_1|$, relative to the fluid particle velocity:

$$P_1 = |F_1| \sin\varepsilon_1$$

$$Q_1 = -|F_1| \cos\varepsilon_1 \qquad (20)$$

and:

$$\varepsilon_1 = \tan^{-1}\{P_1/(-Q_1)\} \qquad (21)$$

Figure 12 indicates the variation with A/D of the values of ε_1 computed from the data of Reference 26. The results are consistent with those in Figure 8. For A/D above 2 the force leads velocity by about 15^o and thus in this range Q_1 is about 96%, and P_1 is about 25% of $|F_1|$. Below an amplitude ratio of unity the force leads velocity by about 35^o and the corresponding ratios are 82 and 57%. Whether or not there is a real transition in the behavior of the component in phase with acceleration between these two amplitude values is clouded somewhat by the fact that data for low values of A/D and those for high were not obtained at comparable non-dimensional frequencies. Since the Keulegan[26] data for P_1 is, for practical purposes, all that exists, it is difficult to see how to sort this out much further.

Returning to the consideration of the third harmonic component, the question arises as to whether or not this component is superimposed on the fundamental force component in a systematic way. The data in hand from Reference 26 allows the computation of Fourier coefficients for the case that the time scale of Equation 18 is shifted by an arbitrary increment, δt, such that:

$$t = t' + \delta t.$$

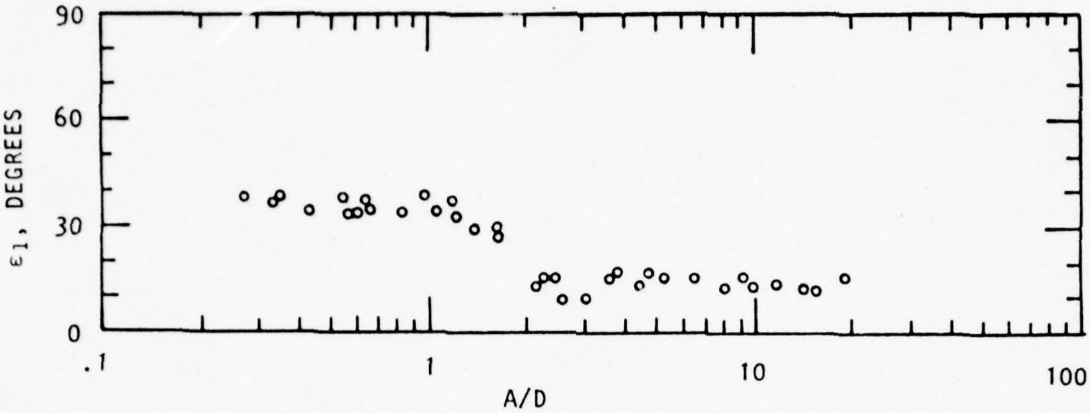

FIGURE 12 PHASE LEAD OF FUNDAMENTAL COMPONENT OF FORCE ON PLATES RELATIVE TO FLUID PARTICLE VELOCITY (DATA OF REFERENCE 26)

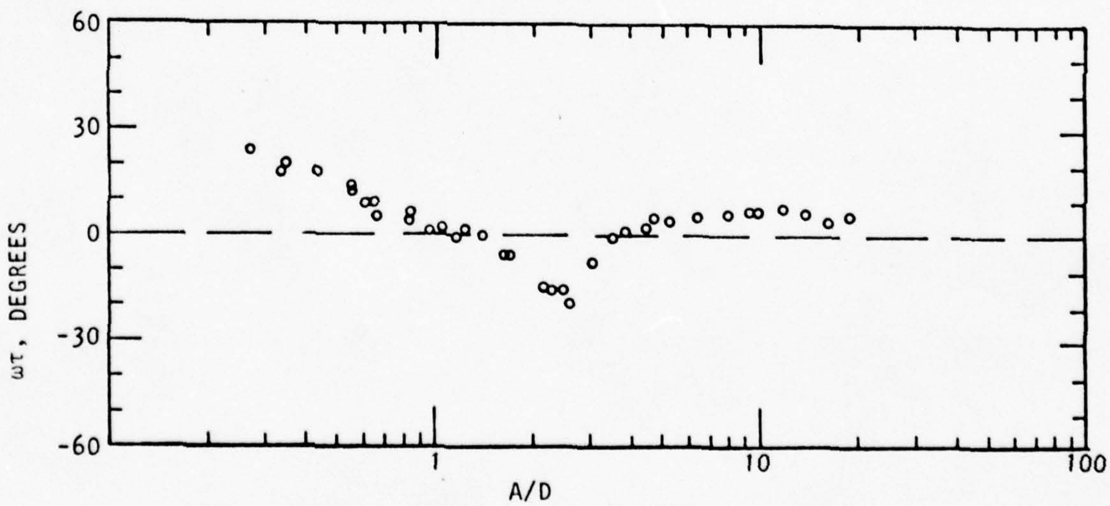

FIGURE 13 PRINCIPAL VALUE OF $\omega\tau$ COMPUTED FROM THE DATA OF REFERENCE 26

If δt is chosen so as to make the resulting coefficient of $\sin\omega t'$ equal to zero, Equation 18 becomes:

$$F = \tilde{Q}_1 \left[\cos\omega t' + \{\tilde{P}_3/\tilde{Q}_1\}\sin 3\omega t' + \{\tilde{Q}_3/\tilde{Q}_1\}\cos 3\omega t'\right] \tag{22}$$

where the fifth harmonic contribution is neglected, and all coefficients on the right hand side are computable from the given data. Relative to $\omega t' = 0$, the maxima and minima in the third harmonic component of Equation 22 occur at times τ corresponding to:

$$\omega\tau = \frac{1}{3}\tan^{-1}\left[\frac{\tilde{P}_3/\tilde{Q}_1}{\tilde{Q}_3/\tilde{Q}_1}\right] + (n\pi/3) \tag{23}$$

where the principal value of the arc-tangent is implied; ($n = 0,1,2...$); and the values of $\tilde{Q}_1$ have not been cancelled in order that the signs of numerator and denominator of the arc-tangent reflect the correct quadrant relative to the expression within large brackets in Equation 22. The total range of the principal value of $\omega\tau$ is $\pm\pi/3 = \pm 60^\circ$. The computations implied by Equations 22 and 23 were carried out with the results indicated in Figure 13. Because $\tilde{Q}_3$ and $\tilde{Q}_1$ are both negative throughout the data of Reference 26, the values of $\omega\tau$ all lie in the range of $\pm 30^\circ$, and as may be seen in Figure 13, the majority of results are within the range of $\pm 10^\circ$. Referring to Equation 22, for the most part the term involving $\tilde{P}_3$ is thus relatively insignificant. This result, with the fact that $\tilde{Q}_3$ and $\tilde{Q}_1$ always have the same sign, shows that the third harmonic component always produces an increase of force magnitude at the instant of maximum (and minimum) fundamental component, and implies that the maximum combined force amplitude should not be far distant from the sum of $\tilde{Q}_1$ and $\tilde{Q}_3$. It is not clear how the trend of the results in Figure 13 comes about, but the fact that there is an apparent trend implies that the phasing of the third harmonic is systematically related to the fundamental component of force, but is not constant.

Numerical values of the ratio of the sum of $\tilde{Q}_1$ and $\tilde{Q}_3$ to $\tilde{Q}_1$:

$$|\tilde{Q}_1 + \tilde{Q}_3|/|\tilde{Q}_1|$$

Have been computed and are shown in Figure 14. Similarly, the peak positive value (F_{mx}) of Equation 22 may be computed, and Figure 14 also indicates the

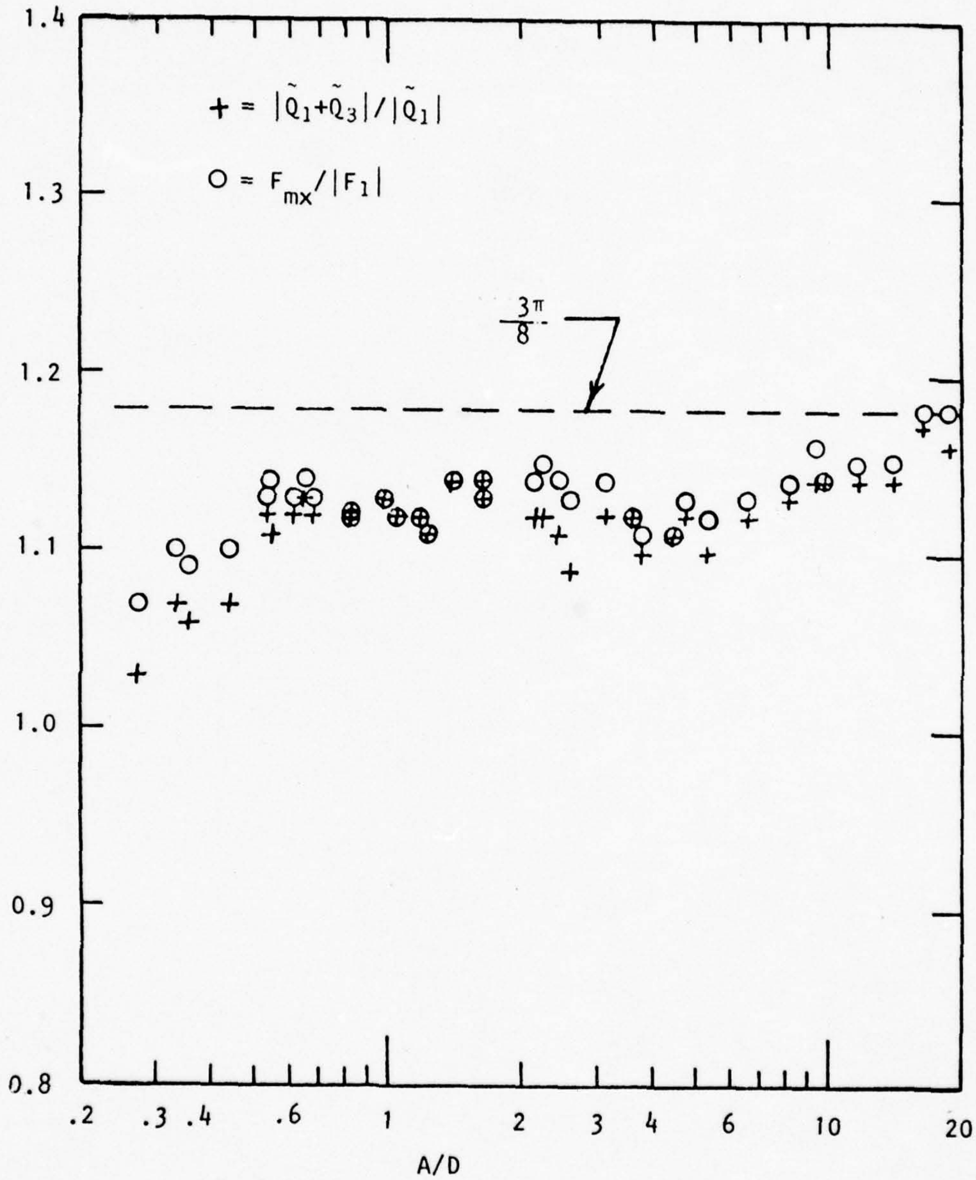

FIGURE 14 $\{|\tilde{Q}_1+\tilde{Q}_3|/|\tilde{Q}_1|\}$ AND $\{F_{mx}/|F_1|\}$ AS FUNCTION OF A/D FOR THE DATA OF REFERENCE 26.

ratio of this computed peak force to the amplitude of the fundamental component. Excepting for low values of A/D, these two ratios generally agree within about 2%, substantiating the inferences just made in the discussion of Figure 13. The discussion of Keulegan and Carpenter's interpretation of the Morison quadratic model which was made in conjunction with Equations 7 and 8 of a preceding section, suggests that an effective amplitude of force should be approximately $3\pi/8$ times the amplitude of the first Fourier compoent. The constant $3\pi/8$ is indicated in Figure 14. The results suggest that this factor is an over estimate, but only by a relatively small percentage.

Figure 14 also indicates the magnitude of error in the data reduction methods involving the use of maximum observed force, or force at the instant of maximum velocity, as estimators of effective amplitude. That the error committed is relatively small seems verified by the data of Reference 26.

CHARACTERISTICS OF FORCE TIME HISTORIES IN THE TIME DOMAIN

In reviewing the literature on plates in oscillatory flow, no serious refutation has been made of the most fundamental qualitative ideas about the character of the force. Perhaps the most fundamental ideas are that, since the flow field is defined to be periodic and symmetrical about zero velocity, the force time history must also be symmetrical about zero and have the same period as the flow field. The consequence is that reasonable mathematical models, for force response having the same period as that of the flow, involve odd terms and/or odd Fourier components. The "quadratic" term in the Morison model ($|U|U$, Equation 5) is odd and also satisfies the periodicity and symmetry ideas. There is only one exception to the periodicity and symmetry idea in the literature on the oscillatory forces on cylinders (Appendix A). This exception occurs in Sarpkaya[56*] where a few of an enormous number of force time histories were apparently found not to satisfy symmetry, in the sense that the mean was not zero. An examination of the one detailed result given in Reference 56 discloses that the time history is also not periodic in a time equal to the period of the flow field. Since in a non-linear system there is no prohibition of sub-harmonic response, there is a possibility that the force could be symmetric over a time equal to an actual longer periodicity. The explanation given in Reference 56 for the non-symmetry involves fractional shedding of vortices not necessarily on alternate sides of the cylinder. Whether or not this phenomenon could happen for plates in oscillatory flow is an open question. No experiments comparable in scope to those of Sarpkaya[56] have been done for plates in oscillatory flow.

Given the periodic, symmetric force assumption for plates, an examination was made in Reference 20 of the relative merits of the Morison model, Equation 5,

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*56. Sarpkaya, T., "Vortex Shedding and Resistance in Harmonic Flow About Smooth and Rough Cylinders at High Reynolds Numbers", Naval Postgraduate School, Monterey, Report NPS-59SL76021, February 1976, AD-A020 029/5ST.

and a straight forward odd Fourier series. The data used was that of Reference 26. The conclusion was that both the magnitude, and position within the periodic cycle, of the peak observed force on plates was reflected very well by an odd Fourier series truncated after the third harmonic term. The correspondence was typically within 2 or 3% for force magnitude and the position of the peak force was predicted within about 1/60 of the oscillation period. Quantitatively, this degree of correspondence was found to be significantly better than that predicted by the Morison model. Equation 5. In the single case in which an observed plate time history was available, the truncated Fourier series represented the observed force time history throughout the cycle with deviations of approximately the estimated resolution of the observed data. The Morison model was clearly inferior in this case; maximum deviations from the observed time history were ±15% of peak force.

With this result from the data of Reference 26 it was of interest to examine other references on plate experiments to see if any other complete force time histories were available. Apart from those given by Keulegan[26], only McNown[34] and Shih[54] (Buchanan[36]) give time histories which are documented as being observations from a flat plate experiment.

As previously noted, the time history data of McNown covers only 3/4 of a cycle and is thus not unambiguously Fourier analyzable. Qualitatively however, the portion of the time histories which are given by McNown[34] resemble that of Keulegan[26].

Buchanan[36] gives two time histories as a series of measured points. One of these (Run 42, Figure 5 of Reference 36) is defined by too few points for a creditable fit of fundamental and third harmonics. The other is sufficiently well defined that a least square sense fit of an odd Fourier series truncated after the third harmonic could be made. Figure 15 indicates the results of this effort. The time and force axes are definedas in References 36 and 54 (Figures 6 and 2 respectively). The circles are the observed digitized points given in the References and the dashed line is a fit (made by Shih and Buchanan in some best sense) of the Morison model to the data. The solid line is the present least squares fit to the data of a Fourier series composed only of fundamental and third harmonic components. Considering that the observed data was measured from oscilloscope photographs, the differences between the

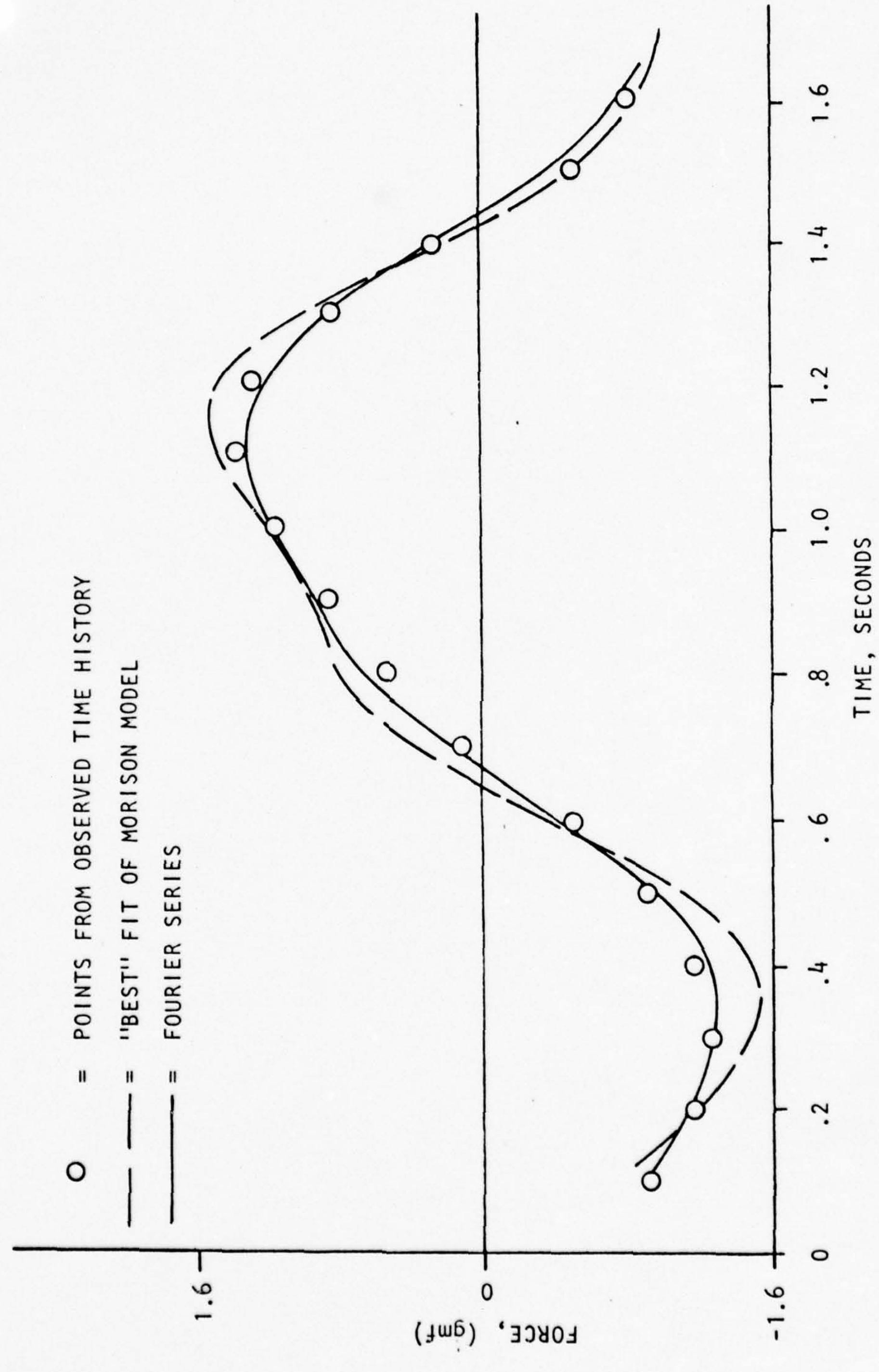

FIGURE 15 COMPARISON OF MEASURED AND COMPUTED FORCES ON A PLATE
(RUN 12, FIGURE 2 OF REFERENCE 54)

observations and the Fourier series result may be of the order of the resolution of the data. In this case the Morison model also represents the observations less well than the truncated odd Fourier series.

Figure 13 helps explain the differences in the representation of time histories. The Keulegan[26] data shows that the "phase" of the third harmonic relative to the fundamental varies with amplitude ratio. The corresponding "phase" of the third harmonic of the Morison model is fixed relative to the component in phase with velocity. Most fitting procedures for the Morison model tend to insure correspondence of computed and observed peak force magnitudes at the expense of the representation of the times of occurence of the peaks. If, in an engineering application, all that is of interest is peak force magnitudes there is relatively little to choose between the alternatives. In the sense of the representation of the variation of force with time, the Morison model appears to create deviations from observation rather than explain them.

SUMMARY AND DISCUSSION

Restated, the general objective of the present work was to look for alternatives to the conventional time domain model for the oscillatory forces on plates, using what data could be found in the literature. Attention was paid only to the most fundamental flow situations; plates being oscillated in a direction normal to their plane, or the kinematically equivalent situation where the fluid oscillates. Within these restrictions, the "literature" on plates in oscillatory flow reduces to a relative handfull of references, the most informative of which is practically the oldest (Keulegan and Carpenter[26]), and the youngest of which was published in 1971. In the course of addressing the available literature, attention was also paid to the literature on cylinders in oscillatory flow, since the interest level in this case is currently higher than in the plate case. It appears from the theoretical and experimental literature surveyed for both cases that no viable alternative to the "Morison" or quadratic model has been developed; nor in most cases even considered. The shortcomings of the model are more often discussed in conjunction with the case of cylinders (See Sarpkaya[28,29], McNown[34], Dalton[32]). There appear to be no new practical theories which hold promise of changing the situation.

Another characteristic common to the experimental literature fitting within the present scope of interest is that the oscillatory motion of the plate or cylinder (or motion of fluid) is assumed to be harmonic (sinusoidal or cosinusoidal). There is no literature on more complicated oscillatory motion. Thus all that may be deduced from existing experiments must be done on the basis of this experimental situation.

To the extent possible in a field in which the basic presumptions have been common for decades, the philosophy of the present approach has been to try to let the data indicate what trends and functional dependencies it would. The first step in the procedure adopted was to produce an ignorant dimensional analysis; "ignorant" because no presumptions about the relationship of force to the independent variables was made--only that the problem involved sinusoidal motion of a plate of given width in a fluid of given mass density and kinematic viscosity. Under these circumstances the

dependent and independent variables of the problem are made non-dimensional by parameters which are constant for any particular experiment, and the independent variables themselves are just non-dimensionalized versions of amplitude and frequency of motion.

Correlation of results of the various investigations on plates was sometimes made speculative for several reasons:

1. Half of the available experiments involve finite aspect ratio plates, half the two dimensional situation.

2. Only one reference (Keulegan[26]) contains significant analyzable data on the added mass part of the conventional model.

3. Edge thickness to plate width ratios vary widely.

4. A wide range of frequency variation is available for only two ratios of amplitude to plate width.

5. Very small amplitudes (less than plate width/20) are associated only with finite plate aspect ratios and high frequencies.

6. One important reference involved a splitter plane, and another, which involved a forced oscillation technique, evidently did not correct data for real inertia of the plate model.

From an experimentalist's viewpoint the first (and sometimes most important) thing which must be done is to examine the general character of the data in the time domain. In general, if the system being given cosinusoidal excitation is non-linear, than a range of possible oddities in the response must be anticipated. These include higher or sub-harmonic content, jumps, etc. In the present case it is difficult to envision how a force which is an explicit function of displacement amplitude alone can arise, so that instabilities causing sub-harmonics and/or jumps do not seem highly probable. In the event, no reference where force was actually measured mentions anything of this nature. So far as current state of knowledge is concerned the force response of a plate given sinusoidal motion is periodic in the period of the motion. There also being no evidence to the contrary, the force is also accepted to be symmetric about zero, so that representation of the force by an odd Fourier series is an acceptable idea.

To the limited extent that such a fit has actually been carried out, the odd series approach reflects the detail of time domain measurements to within basic data resolution. The conventional time domain model (Equations 5 or 7) reflects peak positive and negative forces to acceptable engineering accuracy but does not accurately reflect what lies between. In fact it appears that the fundamental and third harmonic are all that is required to reflect the time domain forces to within about 2%; the fifth harmonic amplitude appears to be of a magnitude near the lower limits of normal oscillographic resolution. One reason why the conventional model does not accurately reflect the time domain data is that it was evidently assumed that there were no non-linear effects upon the force component in-phase with acceleration (the first term of Equation 5). Higher harmonics were assumed to arise only from the velocity non-linearity in the second term. Keulegan and Carpenter's work[26] showed this to be untrue.

Given the foregoing considerations it can be expected that for any given combination of amplitude and frequency there exists a fundamental component of the force which can be resolved via Fourier analysis into components in-phase with acceleration and with velocity. Had any of the experimentalists involved not had prior guidance about how these component forces should vary with the independent variables (amplitude and frequency) he would naturally have just plotted the raw force component data in such a way as to display its variation as a function of two variables so that some judgements about functional dependence might be made. This "ignorant experimentalist" approach was adopted in the present effort, under the assumption that the results of dimensional analysis correctly compensated for the effects of plate size and fluid. It was found possible to make reasonable numerical estimates of non-dimensional component forces from the reported averaged "mass" and "drag" coefficients customarily reported.

There was a small amount of data with which the correctness of the basic non-dimensionalization could be directly evaluated. Non-dimensional force components in-phase with velocity for the same non-dimensional frequencies and amplitudes correlated reasonably well for a few experiments with different plate widths and for a few data points where the fluid was effectively different.

Undoubtedly by the purest chance, there were five sets of two dimensional plate experiments where data could be extracted for amplitude to plate width ratios of 0.75, 0.33 and 0.25. Altogether these data spanned nearly 7 decades of non-dimensional frequency and it was possible to see that, above a relatively low "critical" non-dimensional frequency, the force component in-phase with velocity was closely proportional to the square of non-dimensional frequency. Two dimensional data for other amplitude ratios corresponded to frequencies above the critical, and these data too, appeared to be closely proportional to the square of non-dimensional frequency. Though the available three dimensional data was more limited in this respect, the non-dimensional force component in-phase with velocity appeared also to vary as non-dimensional frequency squared for constant amplitude ratio.

Because of the nature of the basic non-dimensionalization, a squared variation with non-dimensional frequency means that forces are essentially invariant with unsteady Reynolds Number above some critical value. Thus the present analysis bears out the identical conclusion of Shih[54], and extends the demonstration to much higher Reynolds Numbers. The current best estimate of the critical non-dimensional frequency for amplitude ratios in the range 0.75 to 0.25 is $\omega D^2/\nu \simeq 100$. Two things could not be demonstrated directly:

1. That the critical frequency is the same for all amplitude ratios.

2. That the conclusion holds for force components in-phase with acceleration.

In a practical sense, non-dimensional frequencies less than about 100 are quite difficult to achieve if the working fluid is water, the plate width is not miniscule and real frequencies of interest in full size or laboratory Ocean Engineering problems are considered. For this reason it appeared reasonable to attempt further progress by discarding the minority of data for non-dimensional frequencies below 100, and to assume that the remainder was directly proportional to non-dimensional frequency squared. This lead to a second stage non-dimensionalization or correlating modulus for oscillatory forces; that is, division of the original non-dimensional

force by the square of non-dimensional frequency is equivalent to division of the raw (dimensional) force by:

$$\rho\omega^2 D^3 L$$

This produces an alternate non-dimensionalization of force which, according to the previous discussion, should collapse all data for the same plate geometry into a single function of amplitude ratio.

This alternate non-dimensionalization was carried out for all available data. To a remarkable degree (considering the large diversity of experimental techniques, frequencies, etc.) the gambit did what was expected.

Because few have been interested in anything but the force in-phase with velocity, most of the data involves this component. For amplitude to plate width ratios in excess of 1/10 and less than 20 the results imply that for constant frequency this component of force varies closely with the 3/2 power of amplitude; with deviations which are practically a constant <u>percentage</u> regardless of amplitude ratio, and independently of plate geometry. Data corresponding to amplitude ratios less than 1/20 are sparse, and all from three dimensional plate experiments. The trend in this range cannot be confirmed but could be anything between amplitude squared and cubed. There is evidently a break-point near amplitudes of 1/10 or 1/20 of plate width where the trends change, and a very slight indication in the data that the effect of sharp plate edges is to drive the break-point to lower amplitudes. This last is in accordance with the prediction of Tseng and Altmann[41] who were considering the same basic data.

The $\rho\omega^2 D^3 L$ correlating modulus was found to collapse the only available data on force in-phase with acceleration into a single function of amplitude. At face value the resulting function has two branches, each of which follow an approximate 3/2 power law. No data is available for this component for amplitudes less than 1/10 plate width.

A similar correlation was carried out with the third harmonic component data of Keulegan[26]. The amplitude of the third harmonic not only collapses to a single function using the frequency squared modulus, the resulting trend with amplitude appears also to be according to a 3/2 power law. The third harmonic component amplitude apparently is a constant fraction of the

fundamental amplitude, is phased relative to the fundamental in such a way as to augment peak forces, but does not bear an exactly fixed phase relationship to the fundamental component in-phase with velocity.

If it is argued (or assumed)

1. That amplitudes less than about 1/20 plate diameter are not important, or the errors due to an incorrect representation are insignificant, and
2. That, similarly, values of frequency parameter $(\omega D^2/\nu)$ less than 100 are impossible or unlikely:

Then the present analysis suggests a representation for the force response to sinusoidal oscillation of a plate of the following form:

$$F \simeq \rho\omega^2 D^3 L \cdot (A/D)^{1.5}\left\{K_1 \cos\{\omega t - \theta_1(A/D)\} + K_3 \cos\{3\omega t - \theta_3(A/D)\}\right\} \tag{24}$$

where: K_1 and K_3 are constants

and $\theta_1(A/D)$ and $\theta_3(A/D)$ are

relatively weak (nearly constant) functions of A/D
(as for instance in Figures 12 and 13).

All previously discussed data limitations apply in Equation 24, the form is just a synthesis of the gross functional dependencies that all the available data imply. The phase functions are not explicitly defined, the data available with which an explicit representation might be made are very sparse. The squared variation of frequency and the 3/2 power in (A/D) are, however, quite strongly indicated by the data, and the constant K_3 is much smaller than K_1.

One of the possibilities suggested by the initial analysis of Reference 20 was that a functional polynominal representation might be applicable. With respect to real world predictions some of the advantages of this representation were pointed out in the Introduction. It has been shown (Bedrosian[10] for example) that if the input or excitation of a system represented in this way is sinusoidal, then the output is periodic and can

contain all harmonics; and that even harmonics arise from the functionals of even degree. Since the most fundamental symptom of the character of forces on oscillating plates is representability by an odd Fourier series, it follows that the most fundamental characteristics might be characterized by an odd functional polynominal. In order to assess the possibilities of representation of forces on oscillating plates in this way, the functional dependencies shown in Equation 24 must be compared to those which follow from sinusoidal excitation of an odd functional polynominal model. Assuming that the motion of the plate is represented by: $X(t) = A\sin\omega t$ and that the system is represented by an odd functional polynominal (truncated after the fifth degree), the response of the hypothetical system may be shown (Reference 10) to be of the form:

$$
\begin{aligned}
F \simeq & -\cos\omega t \cdot R_e\{AG_1(\omega)+A^3G_3(\omega,\omega,-\omega)+A^5G_5(\omega,\omega,\omega,-\omega,-\omega)\} \\
& +\sin\omega t \cdot I_m\{AG_1(\omega)+A^3G_3(\omega,\omega,-\omega)+A^5G_5(\omega,\omega,\omega,-\omega,-\omega)\} \\
& -\cos 3\omega t \cdot R_e\{A^3G_3(\omega,\omega,\omega)+A^5G_5(\omega,\omega,\omega,\omega,-\omega)\} \\
& +\sin 3\omega t \cdot I_m\{A^3G_3(\omega,\omega,\omega)+A^5G_5(\omega,\omega,\omega,\omega,-\omega)\} \\
& -\cos 5\omega t \cdot R_e\{A^5G_5(\omega,\omega,\omega,\omega,\omega)\} \\
& +\sin 5\omega t \cdot I_m\{A^5G_5(\omega,\omega,\omega,\omega,\omega)\}
\end{aligned}
\tag{25}
$$

In Equation 25 the functions $G_n(\omega,\omega,\cdot\cdot)$ are complex n-dimensional functions of ω alone. The influence of amplitude is contained in the coefficient of each such function.

Now comparing the forms of Equations 24 and 25, it may first be noted that both are in the form of an odd Fourier series, and that the coefficients of each term are composed of more or less separable functions of amplitude and frequency. There is no reason why the value of $G_5(\omega,\omega,\omega,\omega,\omega)$ could not be zero or very small so that the presence of the fifth harmonic in Equation 25 is not disturbing. If frequency is fixed, the amplitude of the third harmonic in Equation 25 is an odd series in A, starting with the cube. On the other hand, the amplitude of the third harmonic in Equation 24, for fixed frequency, varies approximately as the 3/2 power (see Figure 11). There is no way a convincing wide range fit to $(A/D)^{1.5}$ can be made with an

ordinary polynominal in (A/D) if it does not contain the first power. Considering the fundamental component, the situation is similar. For fixed frequency both the amplitude (Figure 11) and the coefficient of $\cos\omega t$ (Figure 7) in Equation 24 vary as the 3/2 power of (A/D). On the other hand, either the fundamental amplitude or the coefficient of $\cos\omega t$ in Equation 25 behave as an ordinary odd polynominal in A. The adequacy of a least square percentage deviation fit of a linear-plus-cubic polynominal to the trend of the data has been shown in Figure 7. These considerations suggest the functional polynominal model not to be a satisfactory approach. It should be mentioned however, that a least square percentage fit such as displayed in Figure 7 and a least square fit are different things. Figure 16 indicates a least square fit of a linear-plus-cubic in A/D to $(A/D)^{1.5}$ in comparison with the 3/2 power function. Linear scales are employed. Though what is inferred from such a figure is almost a matter of taste, it would seem from an engineering standpoint that the functional polynominal model might serve as an approximation to the fundamental component of Equation 24 so long as no more precise model is available. A representation of the third harmonic of Equation 24, good in any sense, seems unlikely.

Leaving off discussion of the functional polynominal, the question arises about alternatives. If the third harmonic component of Equation 24 is disregarded, and the component in-phase with velocity is extracted there results:

$$Q_1 \approx -\rho\omega^2 D^3 L K_1' \,(A/D)^{1.5} \cos\omega t$$

(where the approximate 3/2 power fit to Q_1 shown in Figure 7 has been invoked to eliminate the phase function, $\theta_1(A/D)$).

This can be written:

$$Q_1 \approx -K_1' \,\rho D^2 L\omega (A/D)^{.5} \,\{U_m \cos\omega t\}$$

$$= -K_1' \rho D^{1.5} L\omega^{.5} \{U_m{}^{1.5} \cos\omega t\}$$

Since $(U_m \cos\omega t)^{1.5}$ has a fundamental Fourier component with amplitude approximately 15% greater than $U_m{}^{1.5}$ it is tempting to write for sinusoidal motion:

$$Q_1 \approx -K_1' \rho D^{1.5} L\omega^{.5} \; U^{1.5}$$

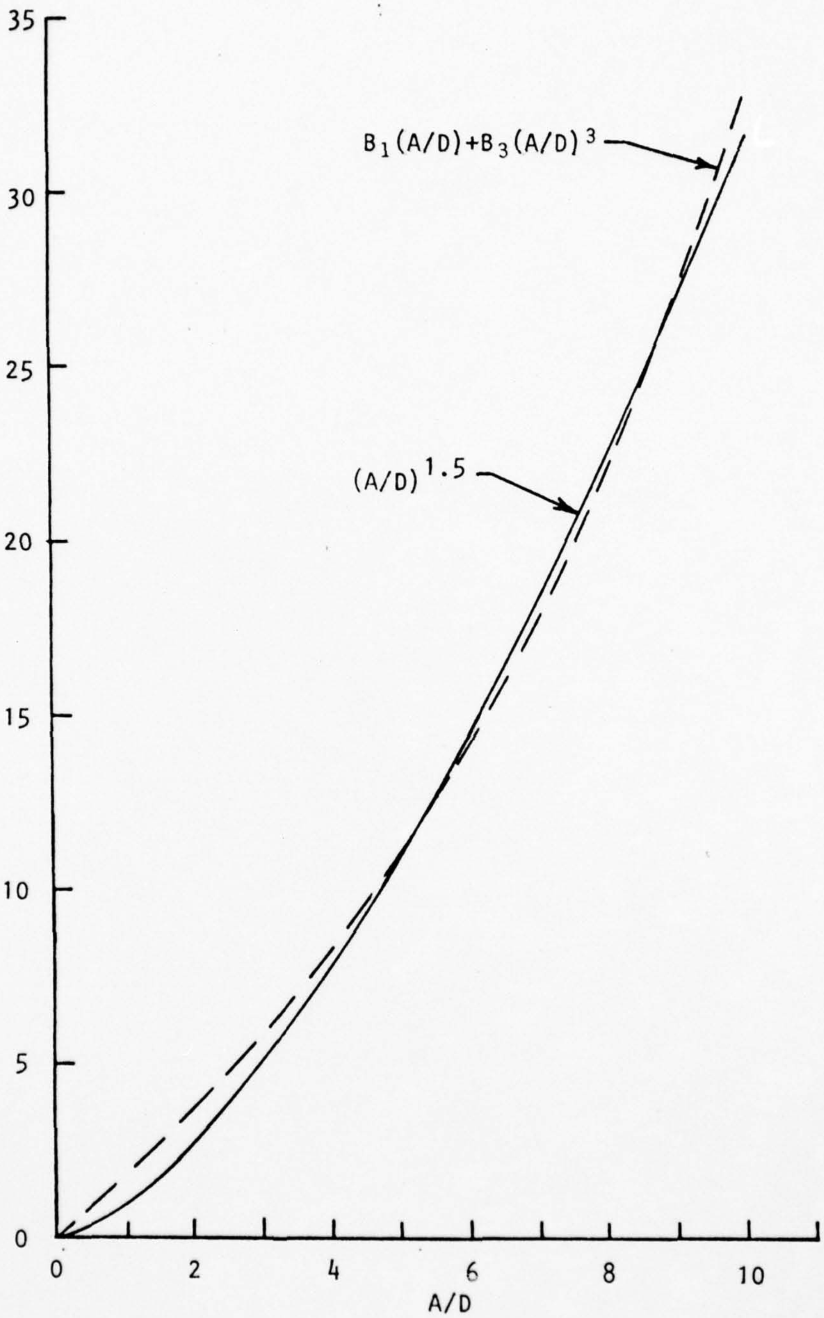

FIGURE 16 LEAST SQUARE FIT OF A LINEAR-PLUS-CUBIC TO A 3/2 POWER FUNCTION

where $U = U_m \cos\omega t$ as in Equation 5. However as far as improving the conceptual model for oscillatory forces is concerned, the above is as much of a nightmare as the conventional model. The coefficient still contains frequency which implies that the system has memory and thus the jump to $U^{1.5}$ is not too justifiable.

CONCLUDING REMARKS

To the extent which information can be extracted from existing empirical data, an alternate to the conventional quadratic model for forces on oscillating plates can be synthesized, but only for sinusoidal motion of the plate. The results of the present work however, do not obviously point the way toward a real-time, non-linear model suitable for more complex forms of motion. Relative to existing data, an approach for which there had been considerable initial hope (the functional polynominal) appears not to be too promising in the scientific sense.

There is considerable danger that the empirical trends and dependencies found suffer from a limited data base. Among the more serious deficiencies from the synthesis point of view are:

1. In absence of an asymptotic theory for large amplitudes (or any theory at all) the connection between available data and the forces for very very large amplitudes is missing, as is the relationship between "steady state" and oscillatory forces. There is a suspicion that the gross functional dependencies developed may not be in the correct direction at the upper end of the experimental range of amplitudes, and thus the danger exists of mis-interpreting the empirical data because the end points are not known.

2. The amount of good information available on the behavior of "mass" or "acceleration" components of force is very small. A demonstration of the degree of influence of viscosity on these components is entirely lacking. Eventual development of a satisfactory time domain non-linear model may depend upon developing an understanding of these components at least equivalent to that in hand for the "drag" or "velocity" dependent components.

3. The present analysis involved the patching together of quite a variety of data because there is so little available that nearly any loss is felt. There is an absence of data, for at least one plate geometry, which covers the complete range of amplitudes and frequencies.

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NON-LINEAR FORCES ON OSCILLATING PLATES: REVIEW AND ANALYSIS OF--ETC(U)

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PRINCIPAL NOTATION

A	Amplitude of motion
C_d	Unsteady drag coefficient
C_m	Unsteady mass coefficient
D	Width of plate or diameter of cylinder
d_j	Fixed geometric parameters
F	Force on a plate or cylinder
F_n	n^{th}. Harmonic of Force F
L	Length of plate or cylinder
P_n	Component of n^{th}. harmonic of force in phase with acceleration
$\underline{\underline{P}}$	Non-dimensional form for P_1
Q_n	Component of n^{th}. harmonic of force in phase with velocity
$\underline{\underline{Q}}$	Non-dimensional form for Q_1
Re	Reynolds Number, steady or unsteady
T_m	Period of oscillation
t	Time
U	Instantaneous velocity $\{=\dot{X}(t)\}$
U_m	Amplitude of velocity
W_m	Amplitude of acceleration
$X(t)$	Displacement of plate or motion of fluid particle
ε_1	Phase angle
ν	Fluid kinematic viscosity
ρ	Fluid mass density
ω	Circular frequency of oscillation

APPENDIX A

DOCUMENTATION OF LITERATURE SEARCH

General

The section on the scope of data of interest in the main part of the report outlines the technical criteria involved. The search documented herein was completed in August 1978. At that time it was thought that diminishing returns had set in and formal search efforts were terminated.

The basic search was conducted both manually and by computer. In the manual part of the search, citations suggested by the Technical Monitor of the project were added to those initially known, and the usual manual strategies were employed from this start. These include working backwards from the references cited in the known and subsequently acquired references, searching in-house files, and scanning the indices of the publications on hand at Davidson Laboratory (which generally cover the Hydromechanic, Naval Architecture and Ocean Engineering Fields).

Data Bases for Automated Search

The computerized literature search system utilized was that run by Lockeed Information Systems (Lockeed Missiles and Space Company, Inc.), is called "DIALOG", and was locally available through the Stevens Institute Library.

Four data bases were utilized at one point or another in the search:

1. SCISEARCH

This is the computerized form of the Science Citation Index (Institute for Scientific Information, Philadelphia, Pa.). It contains about 1.8 million citations from 1974 onward. In addition to authors, titles, etc., all the references noted in each citation are also stored. Thus the data base may be searched for new publications through the subject relationships established by an author's reference to prior work in his field. The effect is to go "forward" in time rather than backward as is the case in manual analysis of an author's reference lists. Key title word searching is also possible.

2. INSPEC

This is the computer version of two of the Science Abstracts family (Institution of Electrical Engineers, London). The two publications included are, Physics Abstracts, and the Electrical and Electronic Abstracts. The data base contains about one million citations starting with 1969. Each citation includes an abstract and identifiers or key words.

3. COMPENDEX

This data base is the machine readable version of the Engineering Index (Engineering Index, Inc., New York). It contains about 0.6 million citations with descriptors and abstracts dating from 1970.

4. NTIS

The NTIS data base is the automated version of the abstracts published by the National Technical Information Service (US Department of Commerce, Springfield, VA). The data base includes abstracts and descriptions for about 0.6 million Government sponsored research and development reports from 1964.

Computer Search Strategy

The search strategy was divided into two parts; citation cross referencing, and key word searching.

The first and most obvious objective was to turn up any recent reference to Keulegan and Carpenters' classic paper (Ref. 26 of the main text) through a search of the Scientific Citation Index. Subsequently, a similar search was made for recent references to eight of the pertinent papers which were known at the outset. These eight references were numbers 1.5, 1.6, 1.11, 1.16, 2.2, 2.4, 3.12 and 3.23 of the appended citation lists (to be subsequently described).

The computer key word search technique allows retrieval of citations in which certain key words or groups of key words occur anywhere in title, abstract or descriptors. The first step is to find, and somehow remember, all citations in which each individually specified key word occurs anywhere. The next step is to combine these results into related groups, say two or three. The last step is then to eliminate all citations

not containing a word from each group.

The initial key-word search strategy was relatively abbreviated. The specific key words and grouping are indicated in Table A-1. These are merely a slight extension of the words in the title of Reference 26. Essentially all citations in the SCISEARCH, INSPEC, and NTIS data bases which contained at least one key word from each group in Table A-1 were listed. It was noted from these results that a few known recent references were not retrieved. The reason was that the key words of Table A-1 were too restrictive, and accordingly, a larger list was made up by analyzing the title words and descriptors of all references in hand at the time. The final key word grouping is indicated in Table A-2. All citations in the COMPENDEX and NTIS data bases which contained at least one key word from each group were listed. (The INSPEC results from the first search had been found to be quite unproductive and this data base was abandoned).

As an illustration of the numbers of citations involved, the final search (Table A-2) of the COMPENDEX data base turned up 26411 citations in Group 1, 22500 in Group 2, and 125184 in Group 3, but the final steps reduced the total number of citations to about 250.

The net result of the searches of the roughly 4 million references in the four data bases previously described was listings of title, descriptors and abstracts (where available) of 660 references. Though half of these had been turned up in the final key word search, the first inspection of the results indicated a large number of repeats of citations occurring in the first computer search and in the manual search, as well as no really new sources of fundamental data on plates. Accordingly, the law of diminishing returns was assumed to be in effect and the computer efforts were terminated.

The First Cut

Since many titles, taken in context, are quite informative, and since abstracts were available for the majority of the 600 references, it was quite straight-forward in the initial review to reduce the total problem to about 90 references a third of which had been known at the beginning.

Magneto-hydrodynamics, plasma dynamics, super-sonic flow, elastic vibration, etc. are all subjects far outside the current range of interest. In many cases where experiments were indicated they clearly involved wave

excitation of objects piercing the free surface, or were experiments involving mean flow of some sort ("struming", oscillations transverse to a mean flow, flow induced pressure fluctuations on buildings, etc.). Many plate experiments dealt with plates at an angle of attack other than 90°. However, where there was the possibility of useful experimental data for present purposes contained in such references, they were retained pending closer examination.

In the 90 remaining references there were several dealing with acoustic streaming flow. Most of these were briefly reviewed in order to learn the special meanings attached to words in this field. In the event, these references were found to be too specialized and all were discarded. Abstracts were not initially available for some of the remaining references, and these were located and scanned. Several additional dropouts occurred, largely on the grounds that the work involved mean flows.

Second Cut

The operations just described reduced the results of the search to 70 references which appeared worthy of examination in more detail. In the process of a somewhat closer examination it became clear that the list could usefully be split up into 5 categories:

Category	References Describing:
1	Experiments on Plates
2	Theory on Plates
3	Experiments on Cylinders
4	Theory on Cylinders
5	References of Indirect Interest

The specific references involved are given in Citation Lists 1 through 5, appended, and these are the basic result of both the manual and computer search procedures.

The fundamental activity in the second cut was deciding which references belonged in category five, the references of indirect interest. As may be noted in Citation List 5, the fifth category may be broken down into two additional parts. Part A consists of 8 references which are in the nature of background, state-of-art reviews, and application papers. Some

of these are equally valuable in providing data source references. The remainder of the 20 references placed in category five (Part B) were principally those about which not enough was known to discard in the first cut. They were examined to see if at least some fragments of data which fit the specifications set forth could be found.

Third Cut

Of the 17 references in Citation List No. 1 for plate experiments, three may be eliminated from further consideration on the basis that nothing closely resembling force on the plate was measured. In this category are Takamatsu, et al (Reference 1.13), Cole (Reference 1.13), and Martin, et al (Reference 1.17). The first two are flow visualization studies, and the last involves measurements of the overall effect of a flat plate on the damping of a U-tube. There were thus finally just 14 references pertaining to experimental work on plates which could be of conceivable use in the present work. None of these was published subsequent to 1971.

A first reading of the items in Citation List 2 indicated the linearized theoretical work of Tuck (Reference 2.3) to be of marginal application in the present case.

In a similar way a first reading of the items in Citation List 3 disclosed several citations which could be eliminated from further consideration. The first of these is Williams (Reference 3.5) where it was belatedly discovered that the cylinder was being oscillated in rotation about its axis rather than in translation. Five references are Masters Theses done under Sarpkaya at the Naval Postgraduate School at Monterey. A close comparison of abstracts, the junior authors, and the acknowledgements of papers and contract reports by Sarpkaya strongly suggested that the experimental results in these Theses are contained in the various references by Sarpkaya. Since the emphasis in the present work is upon plates, not cylinders, it was considered unlikely that acquisition of these five Theses from NTIS would be worth the effort. Accordingly References 3.4, 3.8, 3.10, 3.16 and 3.19 were eliminated from further consideration. Of the 18 remaining references in Citation List 3, 9 are by Sarpkaya, the result of a quite significant series of investigations utilizing two U-tube facilities. The first two references (3.7 and 3.9) involve initial experiments in a small U-tube facility. It is clear from a reading of the succeeding seven

reports by Sarpkaya that this early work was practically superceded by later work in a larger U-tube facility. On this basis there seemed little point in considering these two references in further detail. Of the seven references dealing with the large U-tube experiments two (References 3.11 and 3.15) are the fundamental NSF Grant Reports. The remaining 5 references by Sarpkaya are all based upon the work indicated in References 3.11 and 3.15, the figures used are identical, as is much of the text material; the differences appear almost totally to be that the papers were presented in four different forums. It is thus reasonable to disregard three of the five papers (References 3.13, 3.18, and 3.20). The last two papers (References 3.23, 3.24) are more conveniently available and are excellent summaries of the work. These considerations reduced the list of cylinder experimental references from 24 to 13, (including two references containing both plate and cylinder data).

Of the references in Citation List No. 4, two can be eliminated upon first reading. Reference 4.4 by Chakrabarti is an extremely brief formal discussion which appears not to bear upon the fundamental problem. Reference 4.5 by Hogben contains an extremely intuitive analytical approximation to the relationship between drag and inertia forces and mainly serves to emphasize that this relationship is a vexing and largely unsolved problem.

Summary

Though it is unlikely that any literature search can be absolutely complete, (and the present one is undoubtedly not an exception to the rule) it is thought that the present effort should have turned up at least a lead upon significant fundamental work on the forces on oscillating plates performed since 1971, if such work exists within the English language literature. The fact that it did not suggests that the problem has gone out of vogue. The last spurt of activity was apparently brought on by the problem of controlling rocket booster fuel tank sloshing. There appear to be just 14 experimental references on plates which might conceivably be useful within the constraints placed upon the present work, and practically all of these were known before the formal search effort was initiated.

The recent emphasis on offshore engineering has evidently revived interest in unsteady flow about circular cylinders. However, since the

application involves wave action, the experimental emphasis has been on actual wave experiments rather than the much simpler fundamental situation of current interest. The result in the case of cylinders is also about a dozen references of conceivable utility.

The paucity of purely analytical references suggests that the purely theoretical aspects of the problem have remained as intractable as they were two decades ago when Keulegan and Carpenter completed their experimental work.

TABLE A-1

Key Word Groups, Initial Search

Group 1	Group 2	Group 3
Plate(s)	Oscillat(ory)	Flow(s)
Cylinder(s)	Oscillat(ing)	Liquid(s)
		Water
		Air

TABLE A-2

Key Word Groups, Final Search

Group 1	Group 2	Group 3
Plate(s)	Oscillat(ory)	Fluid(s)
Cylinder(s)	Oscillat(ing)	Liquid(s)
	Unsteady	Flow(s)
	Harmonic	Force(s)
	Period	Damping
	Sinusoidal	Drag
		Resistance
		Wake(s)
		Vortex

CITATION LIST 1

References Pertaining to Experiments which Involve:

Plates Oscillating in a Fluid in a Direction Normal to Their Plane; or Plates in A Fluid which is Oscillating in a Direction Normal to the Plane of the Plate.

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1.3 Tanaka, N. and Kitamura, H., "A Study on the Bilge Keels (Part 2, Full Sized Experiments)" J. of Society of Naval Architects of Japan, Vol. 103, 1958.

1.4 Martin, M., "Roll Damping Due to Bilge Keels", PhD. Dissertation, State University of Iowa, June 1959.

1.5 Cole, H.A. and Gambucci, B.J., "Measured Two-Dimensional Damping Effectiveness of Fuel Sloshing Baffles Applied to Ring Baffles in Cylindrical Tanks", NASA, TN D-694, 1961.

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1.7 Brown, P.W., "The Effect of Configuration on the Drag of Oscillating Damping Plates", Davidson Laboratory, Stevens Institute of Technology, Report 1021, May 1964.

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1.13 Takamatsu, Y., Randall, C.A. Jr., and Dey, S.K., "A Comparative Study of the Flow Field About an Oscillating Flat Plate with the Numerical Solution of the Navier-Stokes Equations", Proceedings of the AIAA/AHS VTOL Research, Design, and Operations Meeting, Georgia Institute of Technology, February 1969, Paper No. AIAA-69-226, AD-686179.

1.14 Cole, H.A., "Effect of Vortex Shedding on Fuel Slosh Damping Predictions", NASA TN D-5705, March 1970.

1.15 Gersten, A., "Scale Effects in Roll Damping", Proceedings of the 16th. ATTC, 1971.

1.16 Shih, C.C., and Buchanan, H.J., "The Drag on Oscillating Flat Plates in Liquids at Low Reynolds Numbers", J. Fluid Mechanics, Vol. 48, Part 2, 1971.

1.17 Martin, S.C. and Bausano, J.C., "Oscillatory Flow Over a Plate Normal to a Wall", Annual ASCE Engineering Mechanics Division Conference: Advances in Civil Engineering Through Engineering Mechanics, Raleigh, May 1977, pp 528-531.

CITATION LIST 2

References Containing Analyses or Reviews Bearing Upon:

Plates Oscillating in a Fluid in a Direction Normal to Their Plane; OR Plates in a Fluid Which is Oscillating in a Direction Normal to the Plane of the Plate.

2.1 Iverson, H. W., and Balent, R., "A Correlating Modulus For Fluid Resistance in Accelerated Motion", J. Applied Physics, Vol. 22, No. 3, March 1951.

2.2 McNown, J.S., and Keulegan, G.H., "Vortex Formation and Resistance in Periodic Motions", Proceedings of ASCE, Engineering Mechanics Division, January 1959.

2.3 Tuck, E.O., "Calculation of Unsteady Flows Due to Small Motions of Cylinders in a Viscous Fluid", California Institute of Technology, Report 156-1, December 1967, AD-829877.

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2.5 Tseng, M. and Altmann, R., "The Hydrodynamic Design of Float Supported Aircraft, I--Float Hydrodynamics", Report 513-5, Hydronautics, Inc., October 1968.

CITATION LIST 3

References Pertaining to Experiments Which Involve:

Circular Cylinders Oscillating in a Fluid in a Direction Normal to their Axis; or Cylinders in a Fluid which is Oscillating in a Direction Normal to the Axis of the Cylinder.

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3.3 Hamann, F.H., and Dalton, C., "The Forces on a Cylinder Oscillating Sinusoidally in Water", ASME Journal of Engineering for Industry, Series B, Vol. 93, November 1971, pp 1197-1202.

3.4 Driscoll, J.R., Jr., "Forces on Cylinders Oscillating in Water", Master's Thesis, Naval Postgraduate School, Monterey, December 1972, AD-758516.

3.5 Williams, R.E. and Hussey, R.G., Oscillating Cylinders and the Stokes Paradox", Physics of Fluids (USA), Vol. 15, No. 12, December 1972.

3.6 Mercier, J.A., "Large Amplitude Oscillations of a Circular Cylinder in a Low Speed Stream", Ph.D Dissertation, Stevens Institute of Technology, 1973.

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3.10 Onur, S., "In-Line Forces Acting on Smooth Cylinders in Harmonic Flow", Master's Thesis, Naval Postgraduate School, Monterey, December 1975, AD-A021830/5ST.

3.11 Sarpkaya, T., "Vortex Shedding and Resistance in Harmonic Flow About Smooth and Rough Cylinders at High Reynolds Numbers", Naval Postgraduate School, Monterey, Report NPS-59SL76021, February 1976, AD-A020 029/5ST.

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3.13 Sarpkaya, T., "In-Line and Transverse Forces on Cylinders in Oscillating Flow at High Reynolds Numbers", Offshore Technology Conference Paper 2533, May 1976.

3.14 Dalton, C., Hunt, J.P., and Hussain, A.K.M.F., "Forces on a Cylinder Oscillating Sinusoidally in Water--2. Further Experiments", Offshore Technology Conference Paper 2538, May 1976.

3.15 Sarpkaya, T., "In-Line and Transverse Forces on Smooth and Sand-Roughened Cylinders in Oscillatory Flow at High Reynolds Numbers", Naval Postgraduate School Report NPS-69SL76062, June 1976, AD-A030403/0ST.

3.16 Collins, N.J., "Transverse Forces on Smooth and Rough Cylinders in Harmonic Flow at High Reynolds Numbers", Master's Thesis, Naval Postgraduate School, Monterey, June 1976, AD-A027247/6ST.

3.17 Yamanoto, T., and Nath, J.H., "High Reynolds Number Oscillating Flow by Cylinders", Proceeding of the 15th. Coastal Engineering Conference (ASCE) Hawaii, July 1976, Chapter 136, pp 2331-2340.

3,18 Sarpkaya, T., "Vortex Shedding and Resistance in Harmonic Flow About Smooth and Rough Circular Cylinders", Proceedings of the Conference on the Behavior of Offshore Structures, BOSS '76, Norway, August 1976, pp 220-235.

3.19 Henning, P.J., "Transverse Forces on Rough Cylinders in Harmonic Flow", Master's Thesis, Naval Postgraduate School, Monterey, March 1977, AD-A035851/5ST.

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3.24 Sarpkaya, T., "The Hydrodynamic Resistance of Roughened Cylinders in Harmonic Flow", The Naval Architect, March 1978, Transactions RINA, Vol. 120,1978.

CITATION LIST 4

<u>References Containing Analyses or Reviews Bearing Upon:</u>

<u>Circular Cylinders Oscillating in a Fluid in a Direction Normal to Their Axis; or Cylinders in a Fluid Which is Oscillating in a Direction Normal to the Axis of the Cylinder</u>

4.1 McNown, J.S. and Keulegan, G.H., (Reference 2.2).

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4.6 Stansby, P.K., "An Inviscid Model of Vortex Shedding from a Circular Cylinder in Steady and Oscillatory Far Flows", Department of Civil Engineering Report No. 77/84, May 1977; Proc Institution of Civil Engineers (London), Vol. 63, Part 2, December 1977, pp 865-880.

4.7 McNown, J.S., and Learned, A.P., "Drag and Inertia Forces on a Cylinder in Periodic Flow", (Written Discussion of Reference 3.22), J. of the Waterway, Port, Coastal and Ocean Division of ASCE, Vol. 104, No. 2, 1978, pp 247-250.

CITATION LIST 5

References of Indirect Interest:

A. Background, Reviews, Applications

5.1 Morison, J.R., et al, "The Force Exerted by Surface Waves on Piles", Petroleum Trans., Vol. 189, 1950, pp 149-157.

5.2 Mair, A.W., and Maull, D.J., "Bluff Bodies and Vortex Shedding - A Report on Euromech 17", J. of Fluid Mechanics, Vol. 45, Part 2, 1971.

5.3 Hogben, N., "Fluid Loading of Offshore Structures, A State of Art Appraisal: Wave Loads", Maritime Technology Monograph No. 1, RINA, 1974.

5.4 "Studies on Developing Accuracy in the Estimation Methods of Ship Propulsive Performance in a Seaway". Report of Ship Research Project No. 161, Japan, March 1976.

5.5 Milgram, J.H., "Waves and Wave Forces", Proceedings of the Conference on the Behavior of Offshore Structures, BOSS '76, Norway, August 1976.

5.6 Hogben, N., "Wave Loads on Structures", Proceedings of the Conference on the Behavior of Offshore Structures, BOSS '76, Norway, August 1976.

5.7 Hogben, N., Miller, B.L., Searle, J.W., and Ward, G., "Estimation of Fluid Loadings on Offshore Structures", Proc. Institution of Civil Engineers, Part 2- Research and Theory, Vol. 63, September 1977, pp 515-562.

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B. Mean Flows, Experiments in Waves, Oscillations Normal to Flow, Hydroelastic Response, Arrays of Cylinders, and Proximity Effects

5.9 Bishop, R.E.D., and Hassan, A.Y., "The Lift and Drag Forces on a Circular Cylinder in a Flowing Fluid", Proceedings, Royal Society of London, Series A, Vol. 277, 1964.

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and Marine Engineers
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1 Sperry Systems Management Division
Sperry Rand Corporation
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2 Southwest Research Institute
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1 Tracor, Inc.
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1 Mr. Robert Taggart
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1 Ocean Engr. Department
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Woods Hole, Mass. 02543

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1 Cornell University
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4 University of California
Naval Architecture Department
College of Engineering
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3 California Institute of Technology
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1 Colorado State University
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1 Florida Atlantic University
Ocean Engineering Department
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1 University of Hawaii
Department of Ocean Engineering
2565 The Mall
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2 Institute of Hydraulic Research
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1 Fritz Engr. Laboratory Library
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5 Department of Ocean Engineering
Massachusetts Institute of Technology
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1 Engineering Technical Reports
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Massachusetts Institute of Technology
Cambridge, Mass. 02139

5 St. Anthony Falls Hydraulic Laboratory
University of Minnesota
Mississippi River at 3rd. Avenue S.E.
Minneapolis, Minnesota 55414
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3 Department of Naval Architecture and Marine Engineering-North Campus
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1 College of Engineering
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Attn: Library

1 National Science Foundation
Engineering Division Library
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Washington, D. C. 20550

1 Applied Research Laboratory
P. O. Box 30
State College, Pa. 16801
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Garfield Thomas Water Tunnel

1 Dr. Michael E. McCormick
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1 Dr. Douglas E. Humphreys (Code-794)
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